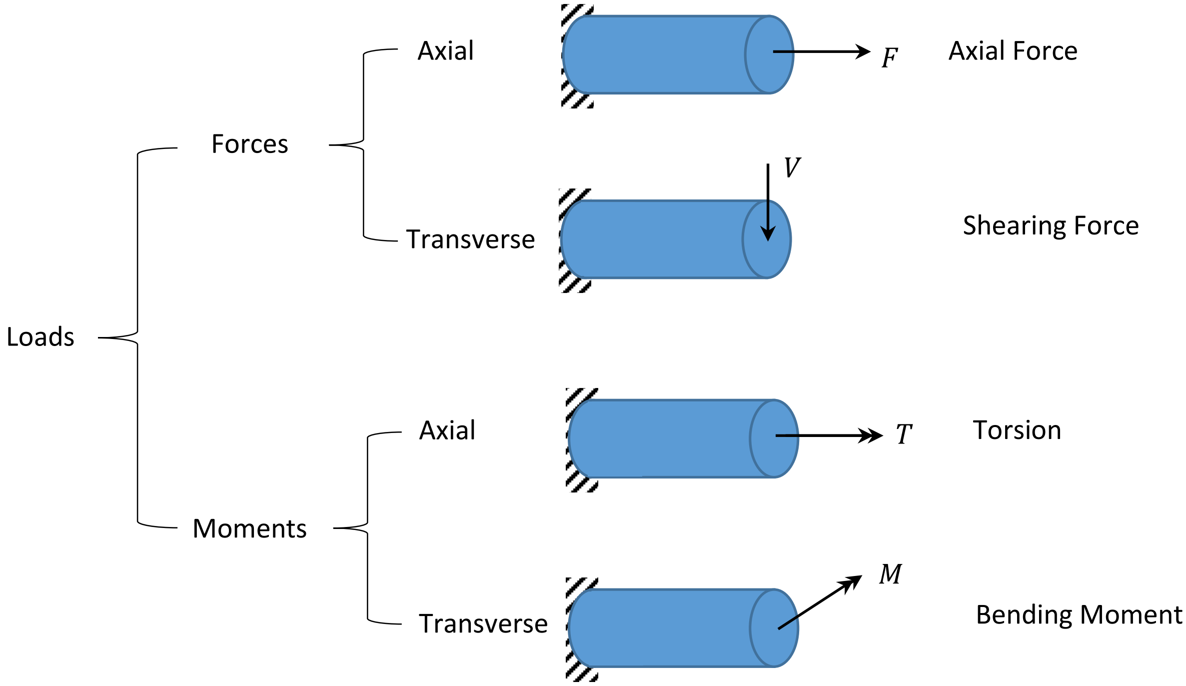
CHAPTER 7-II: INTERNAL LOADS IN BEAMS

MTE-119 Statics

University of Waterloo
Mechanical & Mechatronics Engineering Department

Kamyar Ghavam, PhD
Kamyar.ghavam@uwaterloo.ca

1-1 Loads and Loading Diagrams in Mechanical Members

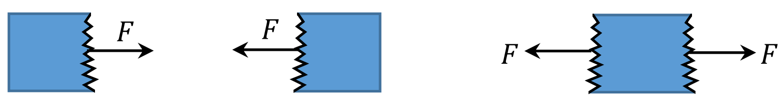


1-2 Axial Force Diagram in Bars

Rule 1: Take unknown internal forces in the positive direction.

Rule 2: Positive forces are tensile and negative forces are compressive.

Rule 3 (Diagram method for axial force): For positive concentrated axial forces move downward in F-X diagram and for negative concentrated forces move upward.



Axial Force Direction Convention - Internal Forces are Action and Reaction

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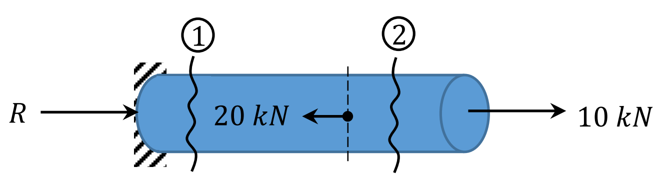
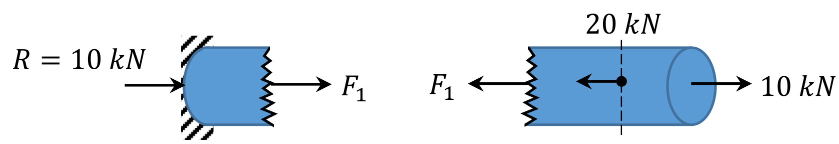
Example 1: Draw the axial force diagram in the following bar, using the “section method” and “diagram method”.



$$R + 10 - 20 = 0 \rightarrow R = +10\ kN$$

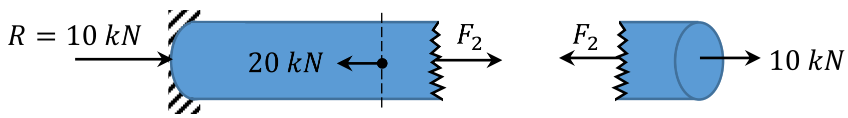
Section 1:



$R + F_1 = 0$ $\quad$ $-F_1 - 20 + 10 = 0$

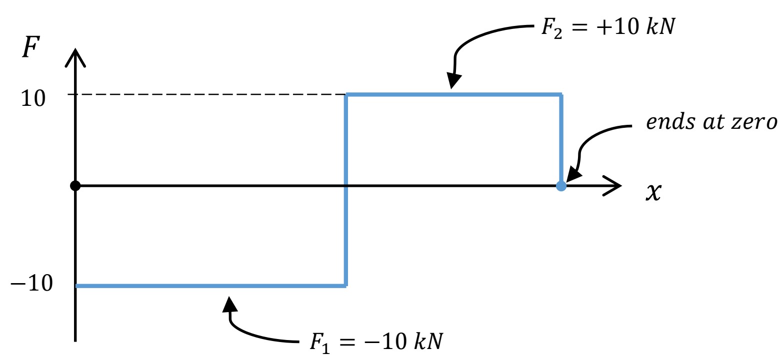
$F_1 = -10\ kN$(Compressive) $\quad$ $F_1 = -10\ kN$ (Compressive)

Section 2:

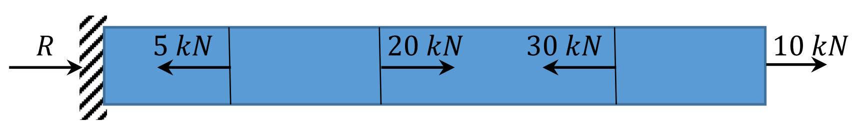


$10 - 20 + F_2 = 0$ $\quad$ $-F_2 + 10 = 0$

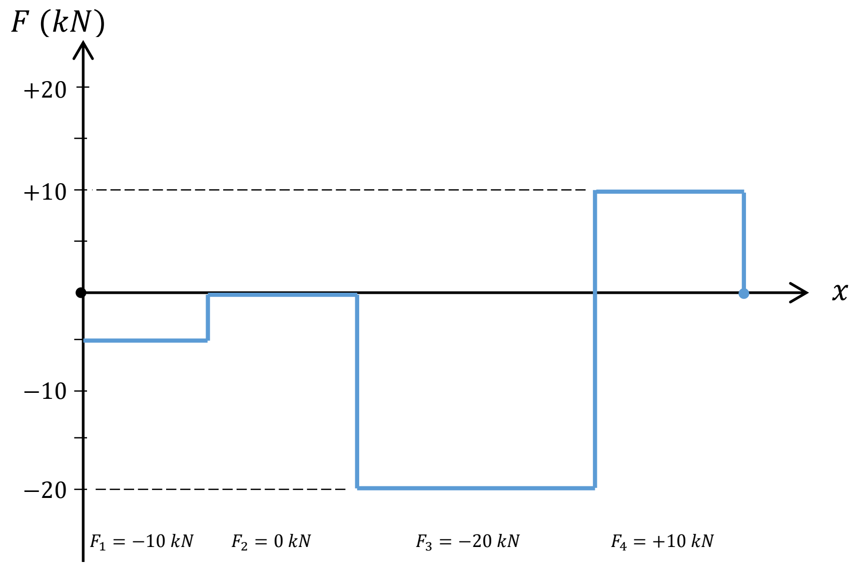
$F_2 = +10\ kN$ (Tensile) $\quad$ $F_2 = +10\ kN$ (Tensile)



Example 2: Draw the axial force diagram using the diagram method.

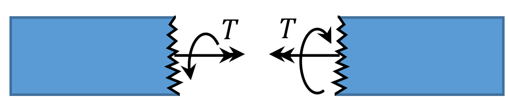


$R - 5 + 20 - 30 + 10 = 0 \rightarrow R = +5\ kN$



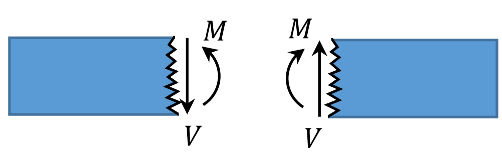
1-3 Torque Diagram in Bars

Rule: It's similar to axial force diagram; for positive torque move downward in T-X diagram and for negative torque move upward.

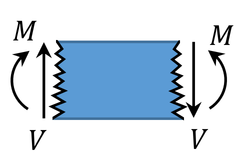


1-4 Shearing Force and Bending Moment Diagram in Beams

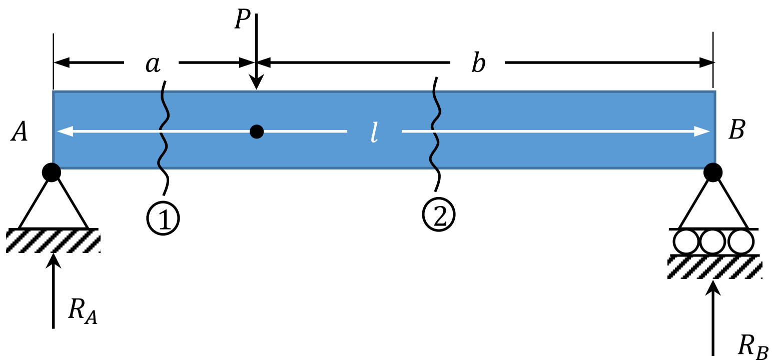
Direction convention for shearing forces and bending moments:



OR



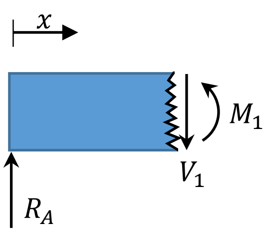
Example 3: Draw shearing force and bending moment diagrams for the following simply supported beam, using the "section" method.



$$\circlearrowleft_+ \sum M_A = 0 \longrightarrow R_B l - Pa = 0 \longrightarrow R_B = \frac{Pa}{l}$$

$$\circlearrowleft_+ \sum M_B = 0 \longrightarrow -R_A l + Pb = 0 \longrightarrow R_A = \frac{Pb}{l}$$

Section 1 $(0 \le x \le a)$:

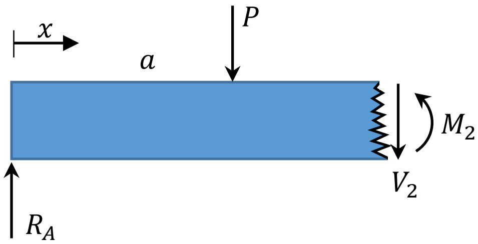


$$\circlearrowleft_+ \sum M = 0 \longrightarrow -R_A x + M_1 = 0 \longrightarrow M_1 = R_A x$$

$$\uparrow_+ \sum F_y = 0 \longrightarrow R_A - V_1 = 0 \longrightarrow V_1 = R_A$$

$$V_1 = \frac{dM_1}{dx}$$

Section 2 $(a \le x \le l)$:

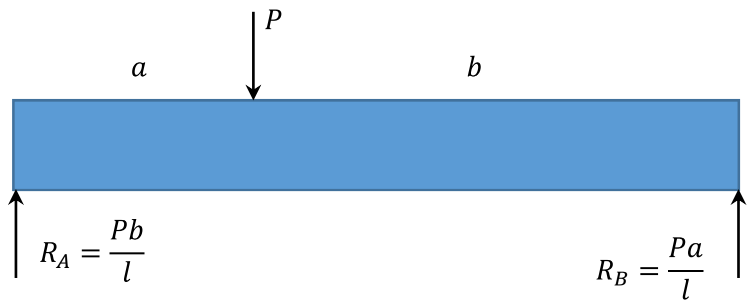


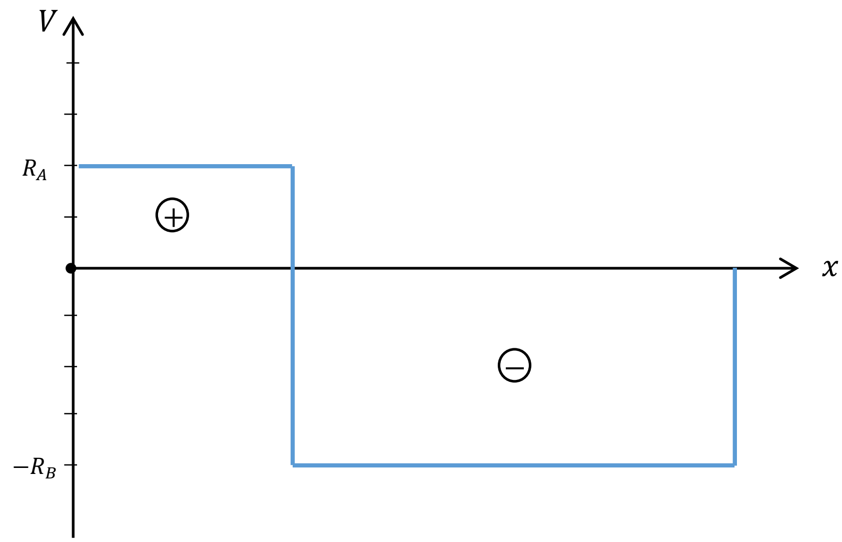
$$\circlearrowleft_+ \sum M = 0 \longrightarrow M_2 - R_A x + P(x-a) = 0$$
$$M_2 = R_A x - P(x-a)$$

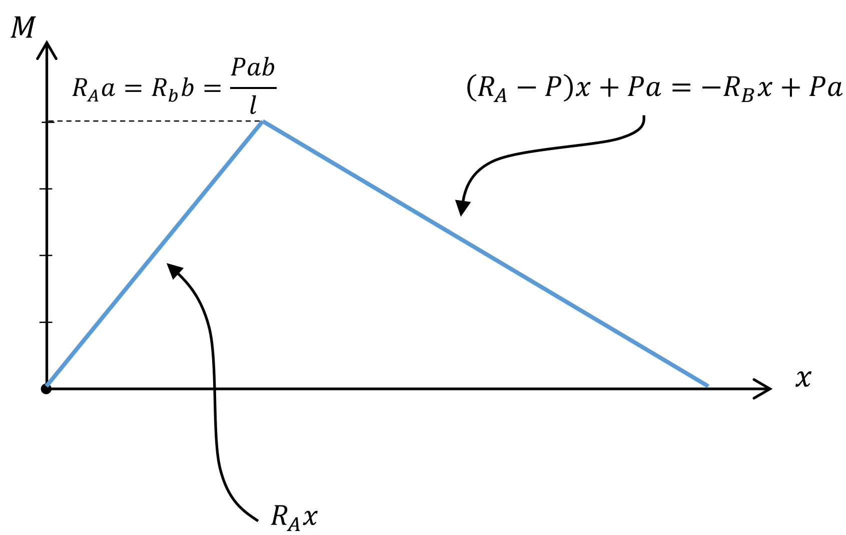
$$\uparrow_+ \sum F_y = 0 \longrightarrow R_A - P - V_2 = 0$$
$$V_2 = R_A - P = -R_B$$

$$R_A + R_B = P \longrightarrow R_B = P - R_A$$

$$V_2 = \frac{dM_2}{dx}$$


P
a
b
$R_A = \frac{Pb}{l}$
$R_B = \frac{Pa}{l}$


V
R_A
$\oplus$
x
$\ominus$
$-R_B$


M
$R_A a = R_b b = \frac{Pab}{l}$
$(R_A - P)x + Pa = -R_B x + Pa$
x
$R_A x$

1-5 Diagram method for drawing shearing force and bending moment diagrams

Rule 1: For positive concentrated transverse (shearing) forces move upward in V(x) diagram and for negative concentrated transverse (shearing) forces move downward.

Rule 2: V at any point represents the slope of M(x) at that point.

$$V = \frac{dM}{dx}$$

Rule 3: Change in M from x_1 to x_2 is equal to the area under V(x).

$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V dx$$

Rule 4: For positive concentrated bending moments move downward in M(x) diagram and for negative concentrated bending moments move upward.

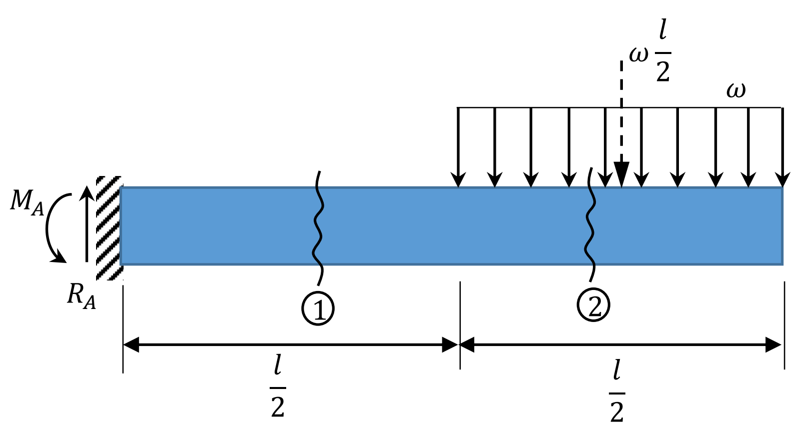
Rule 5: ω at any point represents the negative of the slope of V(x) at that point.

$$\omega = -\frac{dV}{dx}$$

Rule 6: Change in V from x_1 to x_2 is equal to negative of the area under $\omega(x)$.

$$\Delta V = V_2 - V_1 = -\int_{x_1}^{x_2} \omega dx$$

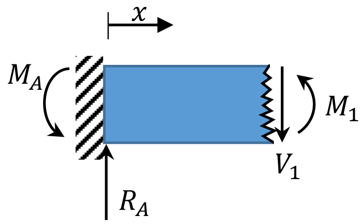
Example 4: Draw shearing force and bending moment diagrams for the following beam, using the "diagram" method.



$$R_A = \omega\frac{l}{2}$$

$$M_A - \frac{\omega l}{2}\left(\frac{l}{2}+\frac{l}{4}\right) = 0 \longrightarrow M_A = \frac{3\omega l^2}{8}$$

Section 1 ($0 \le x \le l/2$):



$$\circlearrowleft_+ \sum M = 0$$

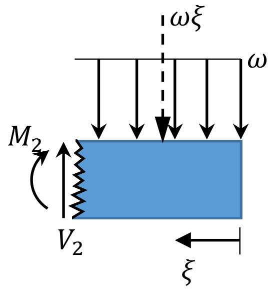
$$M_1 + M_A - R_A x = 0 \longrightarrow M_1 = R_A x - M_A$$

$$\uparrow_+ \sum F_y = 0$$

$$R_A - V_1 = 0 \longrightarrow V_1 = R_A$$

$$V_1 = \frac{dM_1}{dx}$$

Section 2 ($l/2 \le x \le l$):



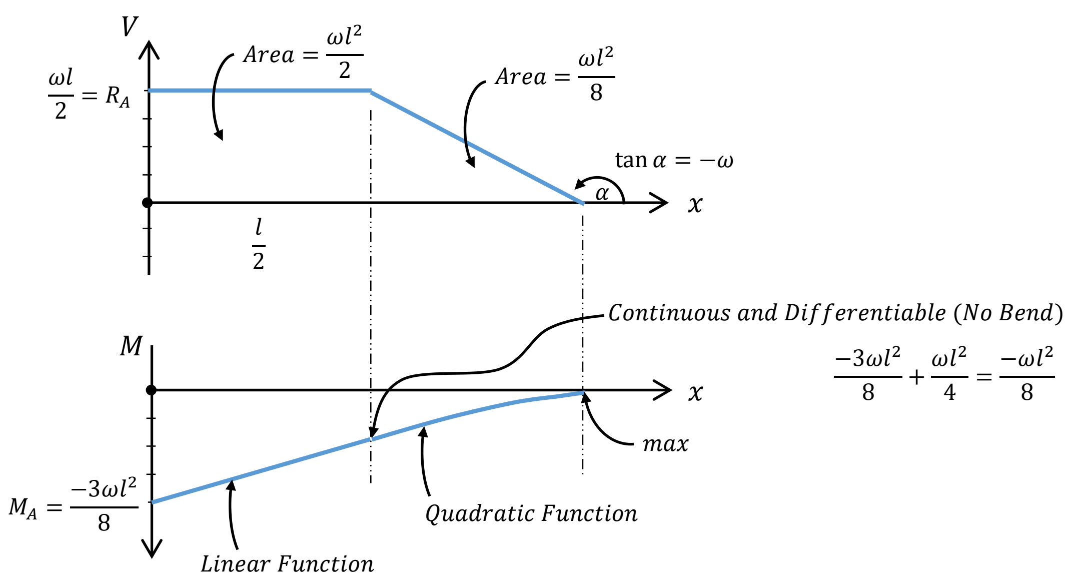
$$\circlearrowleft_+ \sum M = 0$$

$$-M_2 - \omega\xi\frac{\xi}{2} = 0 \longrightarrow M_2 = -\omega\frac{\xi^2}{2} \longrightarrow M_2 = -\omega\frac{(l-x)^2}{2}$$

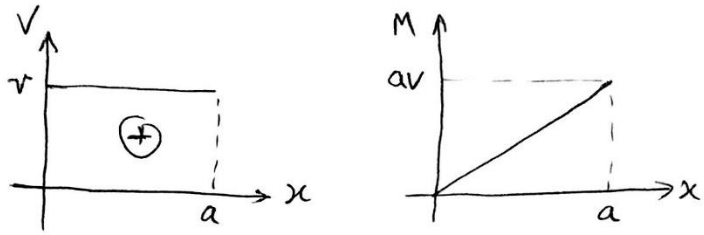
$$\uparrow_+ \sum F_y = 0$$

$$V_2 - \omega\xi = 0 \longrightarrow V_2 = \omega\xi \longrightarrow V_2 = \omega(l-x)$$

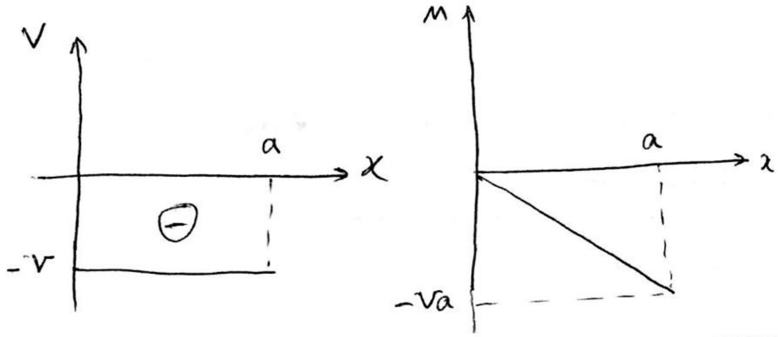
$$V_2 = \frac{dM_2}{dx}$$



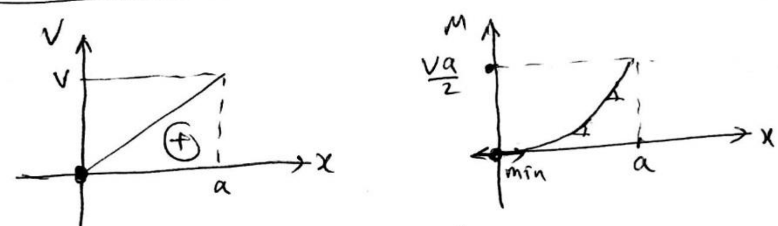
1-a



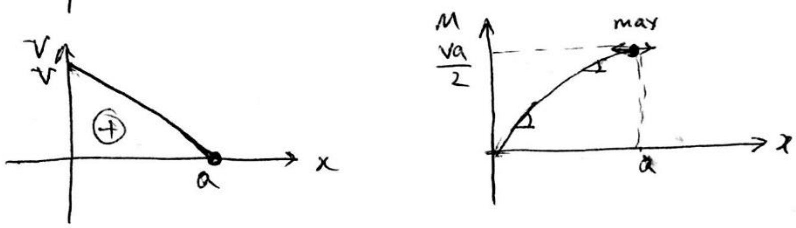
1-b



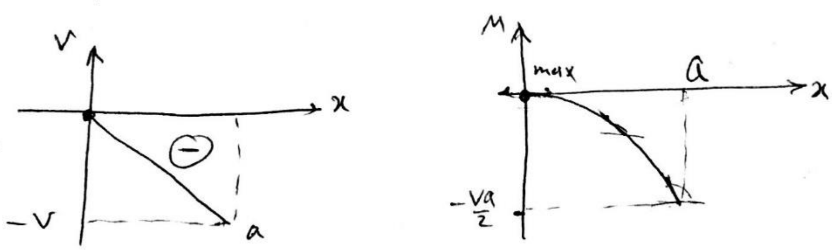
2-a



2-b



2-c



2-d