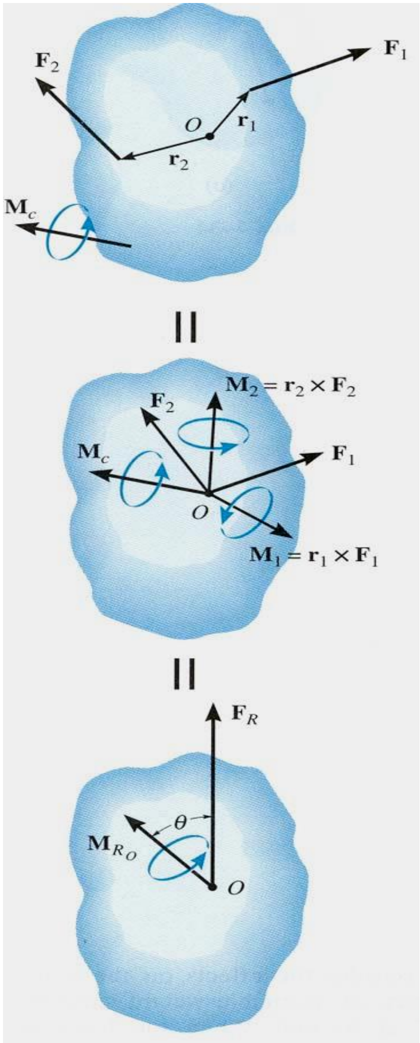
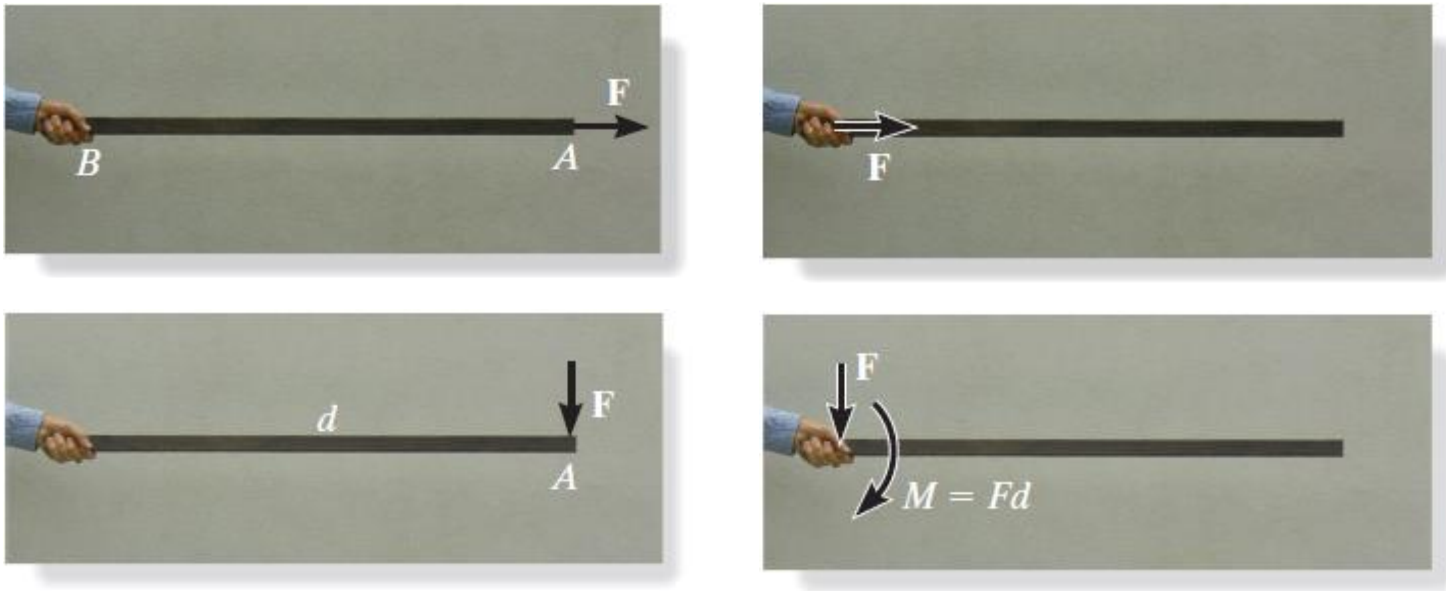
SIMPLIFICATION OF FORCE AND COUPLE SYSTEMS

Objectives:

Find an equivalent force-couple system for a system of forces and couples.



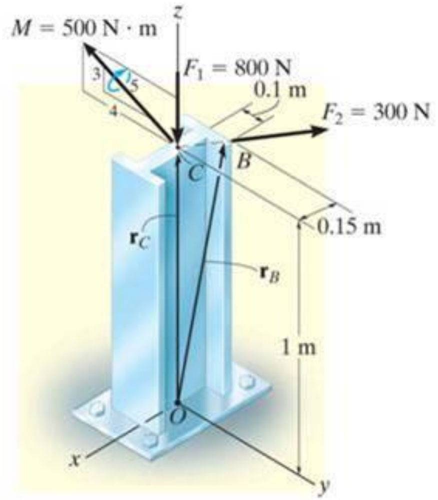
APPLICATIONS



What are the resultant effects on the person's hand when the force is applied in these four different ways?

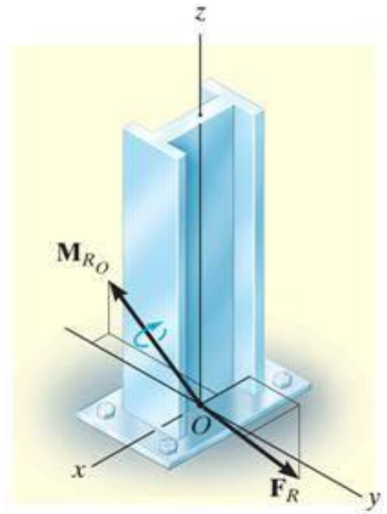
Why is understanding these differences important when designing various load-bearing structures?

APPLICATIONS (continued)



Several forces and a couple moment are acting on this vertical section of an I-beam.

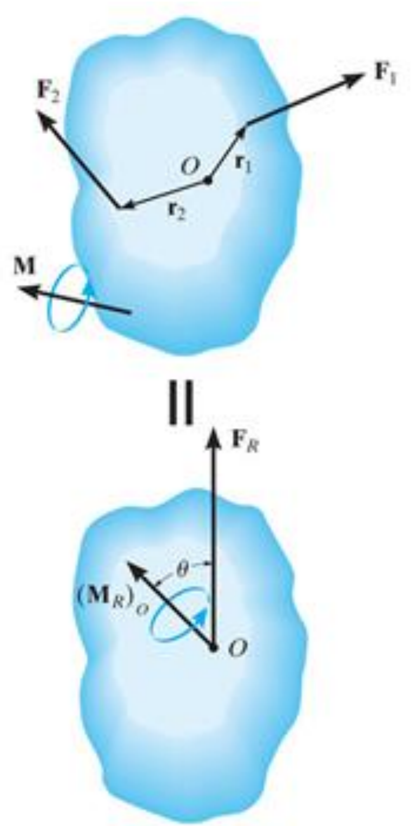
|| ??



For the process of designing the I-beam, it would be very helpful if you could replace the various forces and moment just one force and one couple moment at point O with the same external effect? How will you do that?

PEARSON

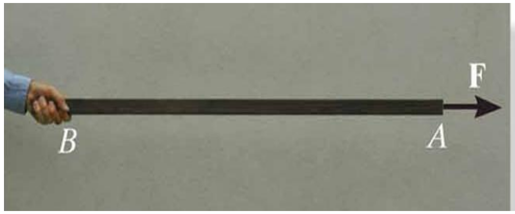
SIMPLIFICATION OF FORCE AND COUPLE SYSTEM

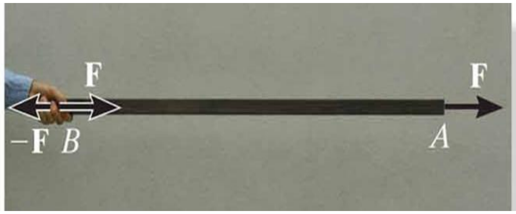


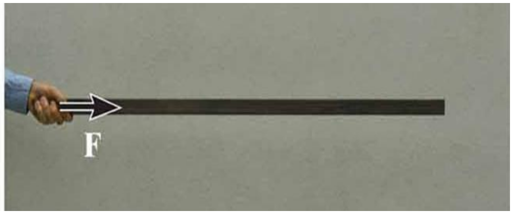
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.

The force and couple systems are called equivalent systems since they have the same external effect on the body.

MOVING A FORCE ON ITS LINE OF ACTION



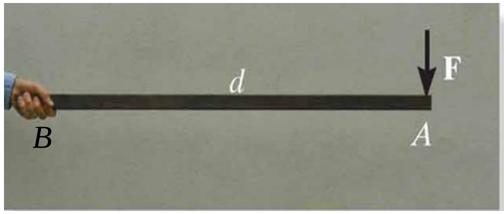


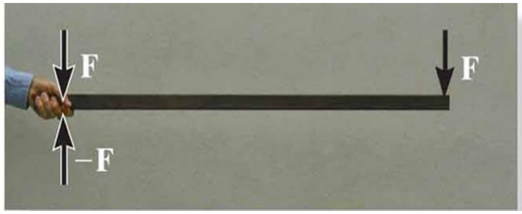


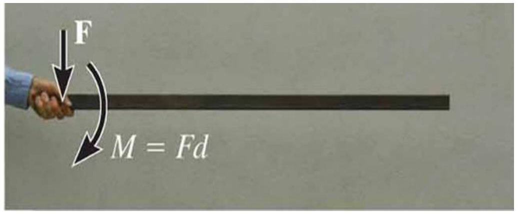
Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect.

Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

MOVING A FORCE OFF OF ITS LINE OF ACTION





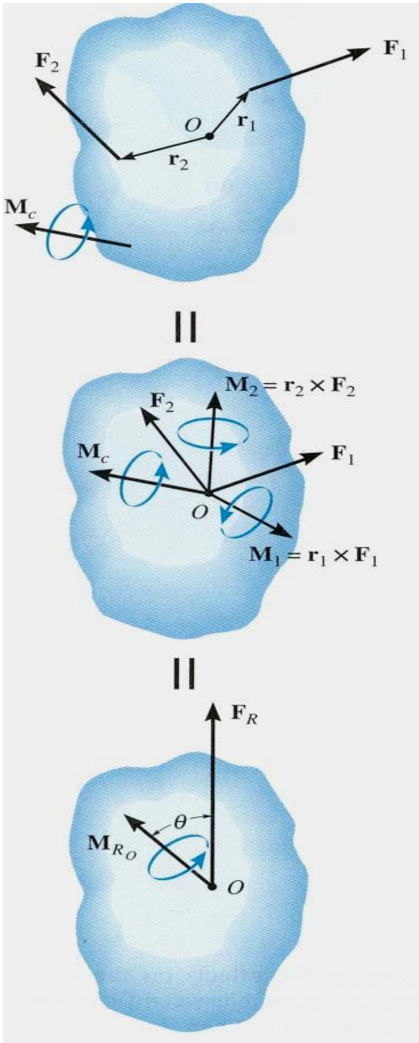


When a force is moved, but not along its line of action, there is a change in its external effect!

Essentially, moving a force from point A to B (as shown above) requires creating an additional couple moment. So moving a force means you have to "add" a new couple.

Since this new couple moment is a "free" vector, it can be applied at any point on the body.

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM



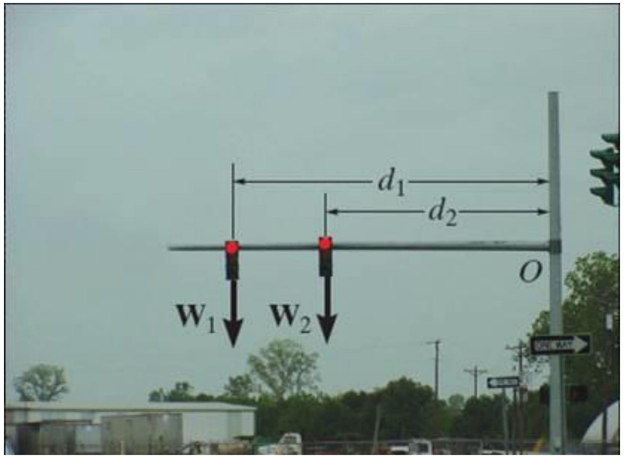
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

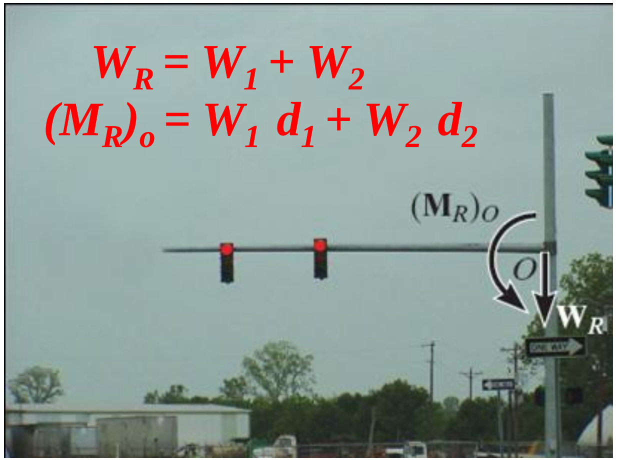
Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (continued)

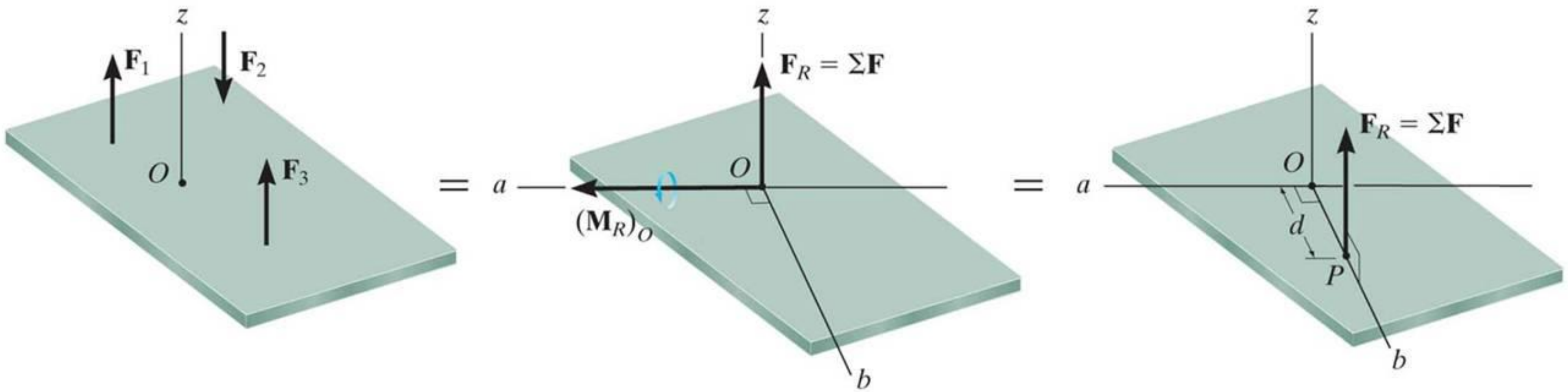




If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

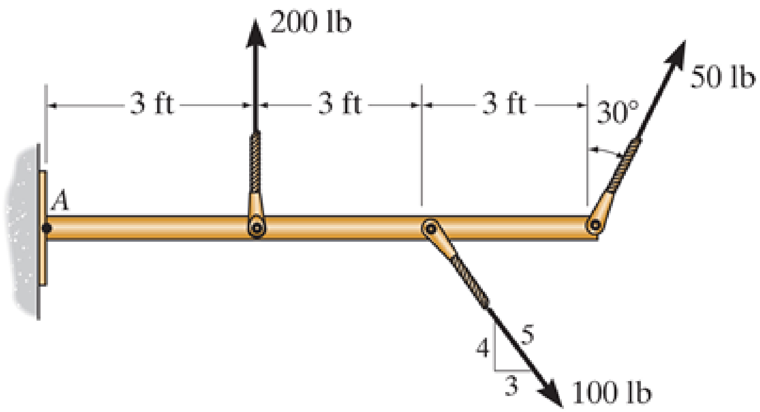
FURTHER SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM



If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R , by simply moving F_R from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

EXAMPLE XI



Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.

Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

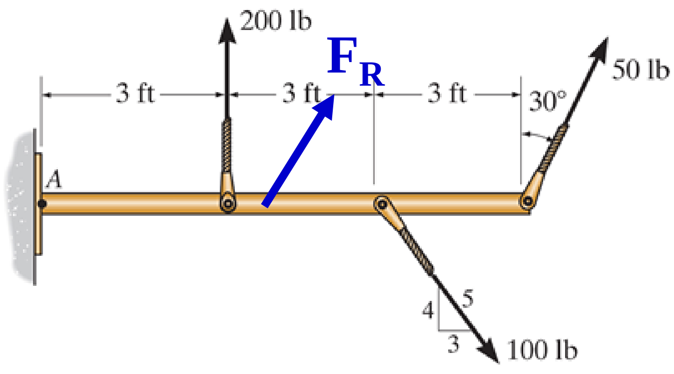
3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE XI (continued)

$+\rightarrow \Sigma F_{Rx} = 50(\sin 30) + 100(3/5)$
$= 85 \text{ lb}$

$+\uparrow \Sigma F_{Ry} = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3 \text{ lb}$

$+\circlearrowleft M_{RA} = 200\ (3) + 50\ (\cos 30)\ (9)$
$-\ 100\ (4/5)\ 6 =$ $\underline{509.7 \text{ lb}\cdot\text{ft CCW}}$



$F_R = (85^2 + 163.3^2)^{1/2} = \underline{184 \text{ lb}}$

$\angle\ \theta = \tan^{-1}(163.3/85) = \underline{62.5°}$

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 509.7 / 163.3 = \underline{3.12 \text{ ft}}$

CONCEPT QUIZ

1. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

   A) One force and one couple moment.

   B) One force.

   C) One couple moment.

   D) Two couple moments.

CONCEPT QUIZ (continued)

2. Consider three couples acting on a body. Equivalent systems will be _______ at different points on the body.

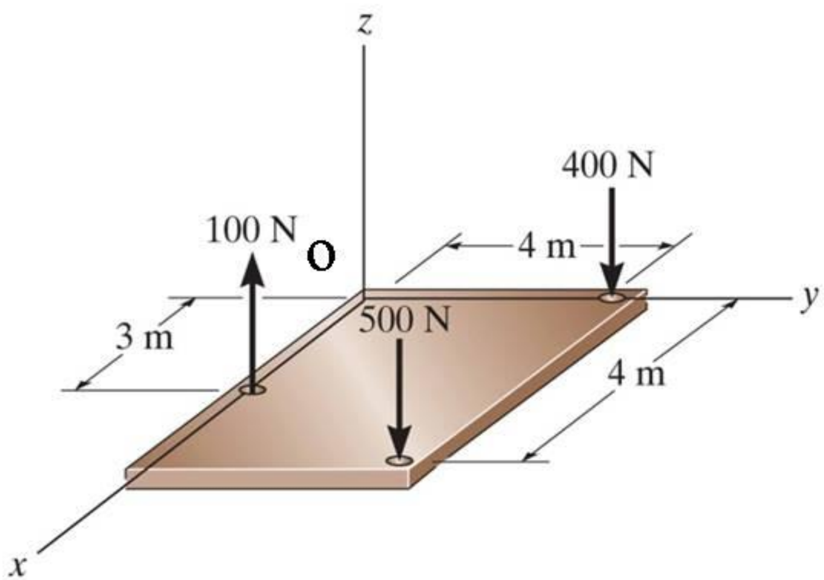
   A) Different when located

   B) The same even when located

   C) Zero when located

   D) None of the above.

EXAMPLE XII



Given: The slab is subjected to three parallel forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force.

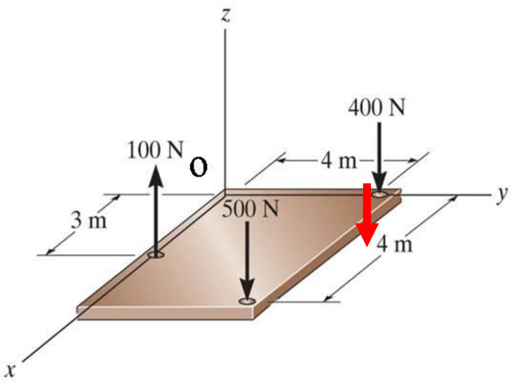
Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i} = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO} / F_{RzO}$ and $y = M_{RxO} / F_{RzO}$

EXAMPLE XII (continued)



$$\mathbf{F}_{RO} = \{100\,\mathbf{k} - 500\,\mathbf{k} - 400\,\mathbf{k}\} = -\,800\,\mathbf{k}\text{ N}$$

$$\mathbf{M}_{RO} = (3\,\mathbf{i}) \times (100\,\mathbf{k}) + (4\,\mathbf{i} + 4\,\mathbf{j}) \times (-500\,\mathbf{k}) + (4\,\mathbf{j}) \times (-400\,\mathbf{k})$$

$$= \{-300\,\mathbf{j} + 2000\,\mathbf{j} - 2000\,\mathbf{i} - 1600\,\mathbf{i}\}$$

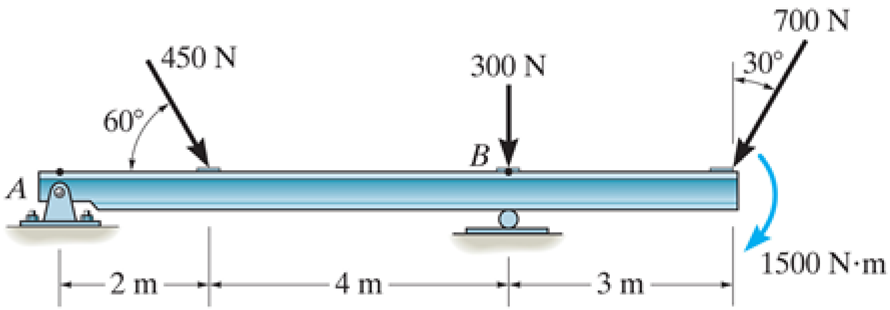
$$= \{\underline{-\,3600\,\mathbf{i}} + \underline{1700\,\mathbf{j}}\}\underline{\text{N}\cdot\text{m}}$$

The location of the single equivalent resultant force is given as,

$x = -\,M_{Ryo} / F_{Rzo} = (-1700) / (-800) = \underline{2.13\text{ m}}$

$y = M_{Rxo} / F_{Rzo} = (-3600) / (-800) = \underline{4.5\text{ m}}$

EXAMPLE XIII



Given: A 2-D force and couple system as shown.

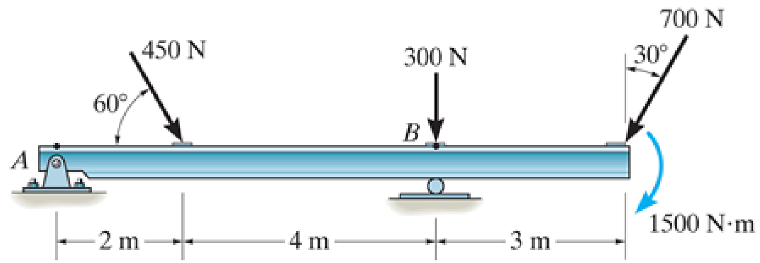
Find: The equivalent resultant force and couple moment acting at A.

Plan:

1) Sum all the x and y components of the two forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to A and add them to the 1500 N·m free moment to find the resultant M_{RA} .

EXAMPLE XIII (continued)

Summing the force components:



$+\rightarrow \Sigma F_x = 450 (\cos 60) – 700 (\sin 30)$

$= – 125 \text{ N}$

$+\uparrow \Sigma F_y = – 450 (\sin 60) – 300 – 700 (\cos 30)$

$= – 1296 \text{ N}$

Now find the magnitude and direction of the resultant.

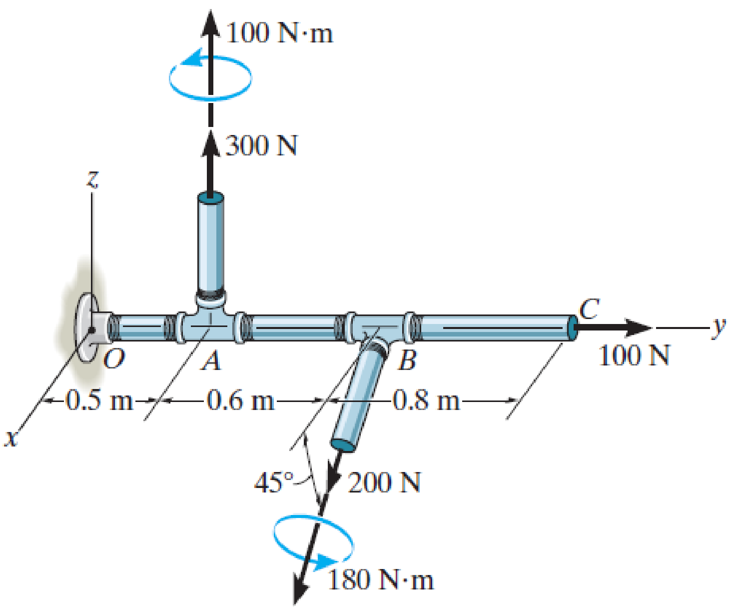
$F_{RA} = (125^2 + 1296^2)^{1/2} = \underline{1302 \text{ N}}$ and $\theta = \tan^{-1} (1296 /125)$

$= \underline{84.5°}$

$+\circlearrowleft M_{RA} = 450 (\sin 60) (2) + 300 (6) + 700 (\cos 30) (9) + 1500$

$= \underline{9535 \text{ N·m}} \circlearrowleft$

EXAMPLE XIV



Given: Forces and couple moments are applied to the pipe.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$

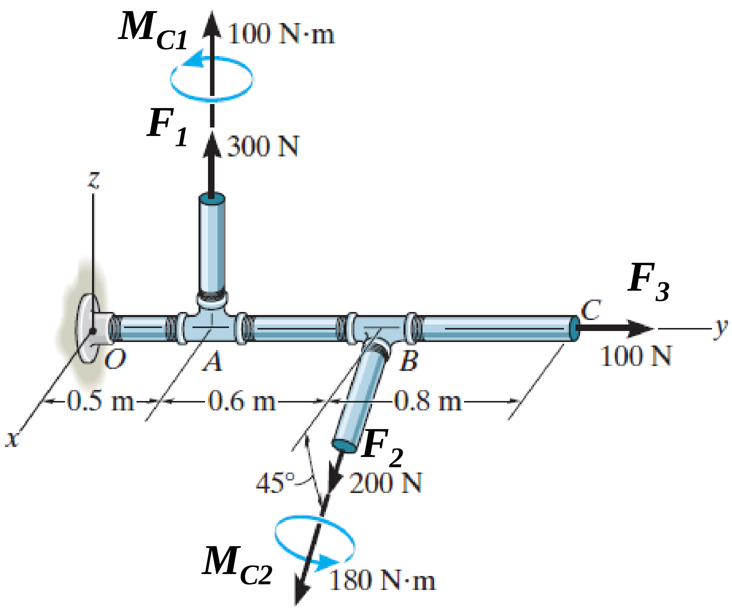
b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\boldsymbol{r_i} \times \boldsymbol{F_i})$

where,

$\boldsymbol{M_C}$ are any free couple moments.

$\boldsymbol{r_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

EXAMPLE XIV (continued)



$\boldsymbol{F_1} = \{300\ \boldsymbol{k}\}$ N

$\boldsymbol{F_2} = 200\{\cos 45°\ \boldsymbol{i} - \sin 45°\ \boldsymbol{k}\}$ N

$= \{141.4\ \boldsymbol{i} - 141.4\ \boldsymbol{k}\}$ N

$\boldsymbol{F_3} = \{100\ \boldsymbol{j}\}$ N

$\boldsymbol{r_1} = \{0.5\ \boldsymbol{i}\}$ m, $\boldsymbol{r_2} = \{1.1\ \boldsymbol{i}\}$ m,

$\boldsymbol{r_3} = \{1.9\ \boldsymbol{i}\}$ m

Free couple moments are:

$\boldsymbol{M_{C1}} = \{100\ \boldsymbol{k}\}$ N·m

$\boldsymbol{M_{C2}} = 180\{\cos 45°\ \boldsymbol{i} - \sin 45°\boldsymbol{k}\}$N·m

$= \{127.3\ \boldsymbol{i} - 127.3\boldsymbol{k}\}$N·m

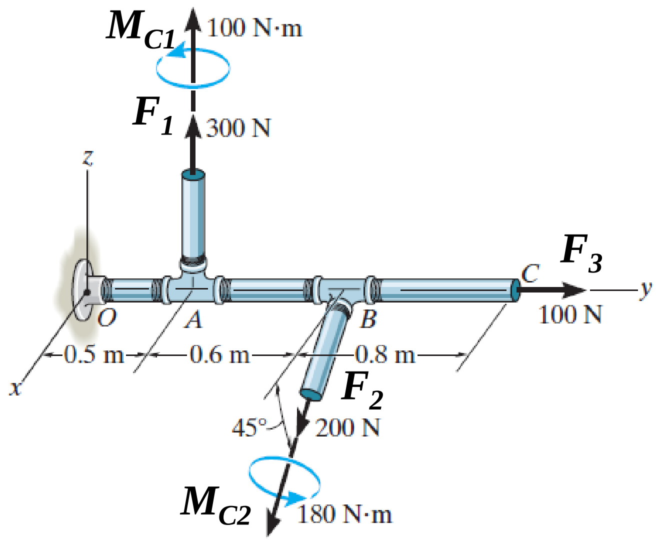
EXAMPLE XIV (continued)

Resultant force and couple moment at point O:

$$\mathbf{F}_{RO} = \Sigma\, \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= \{300\,\mathbf{k}\} + \{141.4\,\mathbf{i} - 141.4\,\mathbf{k}\} + \{100\,\mathbf{j}\}$$

$$\mathbf{F}_{RO} = \{\underline{141}\,\mathbf{i} + \underline{100}\,\mathbf{j} + \underline{159}\,\mathbf{k}\}\ \underline{\text{N}}$$



$$\mathbf{M}_{RO} = \Sigma\, \mathbf{M}_C + \Sigma\, (\mathbf{r}_i \times \mathbf{F}_i)$$

$$\mathbf{M}_{RO} = \{100\,\mathbf{k}\} + \{127.3\,\mathbf{i} - 127.3\mathbf{k}\}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$$\mathbf{M}_{RO} = \{\underline{122}\,\mathbf{i}\ \underline{-\ 183}\,\mathbf{k}\}\ \underline{\text{N·m}}$$