EQUILIBRIUM OF A RIGID BODY IN 2D

<u>Objectives</u>:

You will be able to:

a) Identify support reactions,

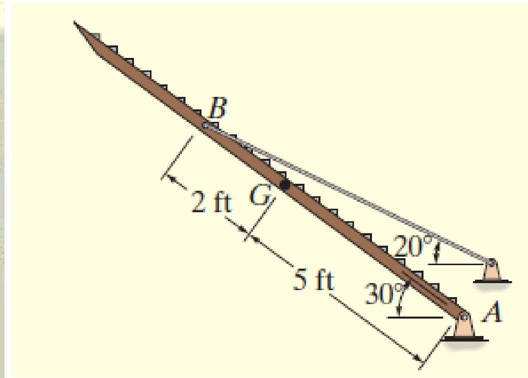
b) Draw a free-body diagrams,

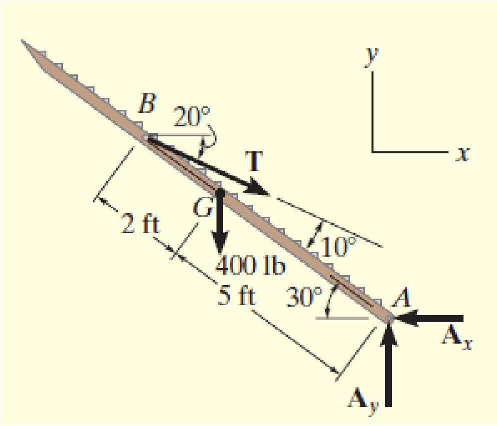
c) Apply equations of equilibrium to solve for unknowns, and,

d) Recognize two- and three-force members.

In **two dimensions**.

APPLICATIONS



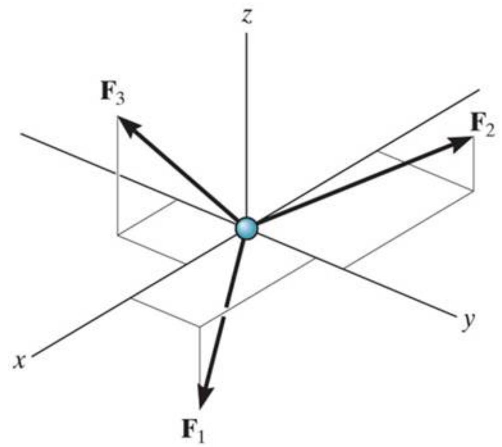


The truck ramps have a weight of 400 lb each.
Each ramp is pinned to the body of the truck and held in the position by a cable. How can we determine the cable tension and support reactions?

How are the idealized model and the free body diagram used to do this?

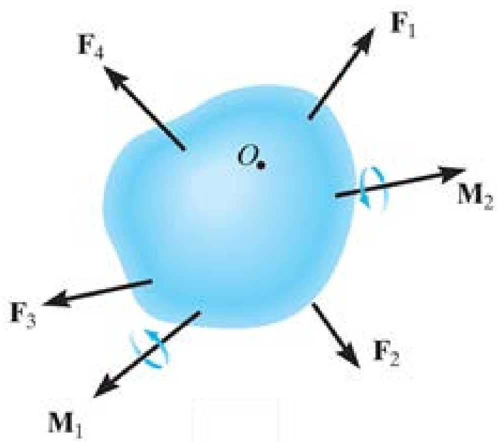
Which diagram above is the idealized model?

CONDITIONS FOR RIGID-BODY EQUILIBRIUM



Forces on a particle

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body (due to moments created by the forces).



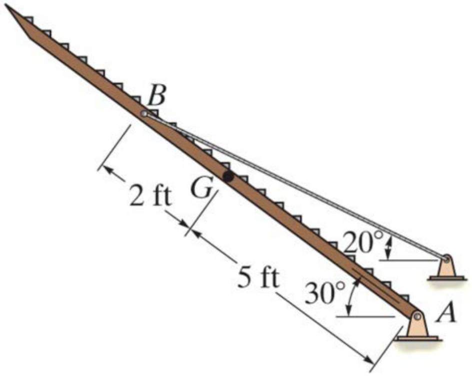
Forces on a rigid body

For a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

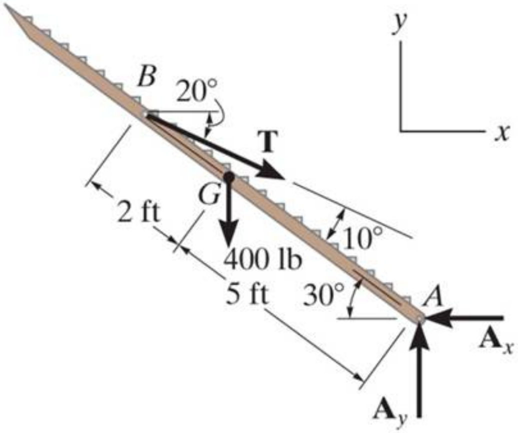
$\sum \boldsymbol{F} = 0$ (no translation)

and $\sum \boldsymbol{M_O} = 0$ (no rotation)

THE PROCESS OF SOLVING RIGID BODY EQUILIBRIUM PROBLEMS



Idealized model

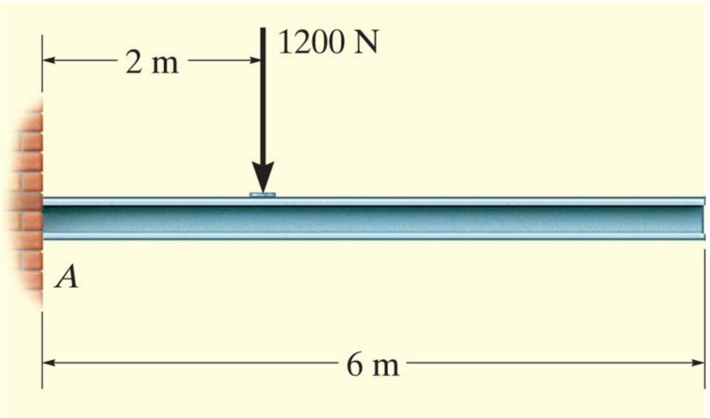


Free-body Diagram

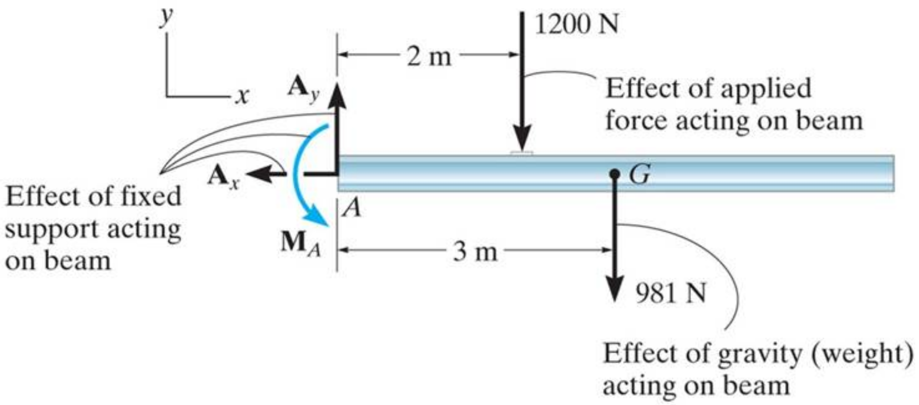
For analyzing an actual physical system, we need to create a free-body diagram (FBD) showing all the external (active and reactive) forces.

Finally, we need to apply the equations of equilibrium to solve for any unknowns.

FREE-BODY DIAGRAMS



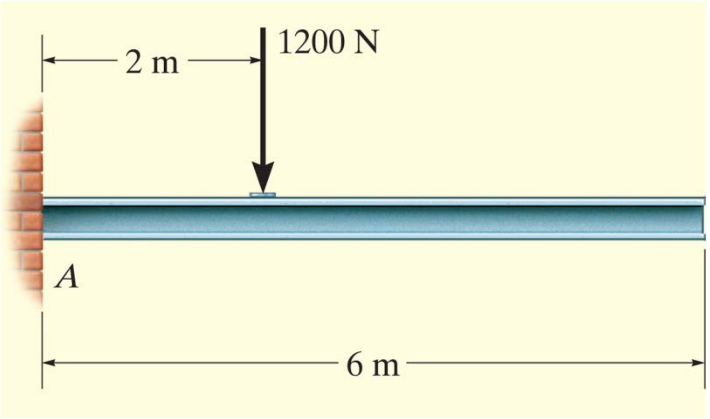
Idealized model



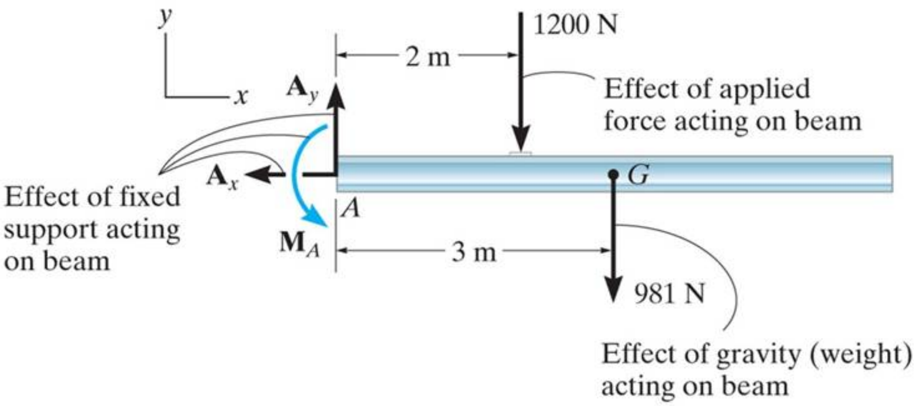
Free-body diagram (FBD)

1. Draw an outlined shape: Imagine the body to be isolated or cut "free" from its constraints and draw its outlined shape.
2. Show all the external forces and couple moments: These typically include: a) applied loads, b) support reactions, and, c) the weight of the body.

FREE-BODY DIAGRAMS (continued)



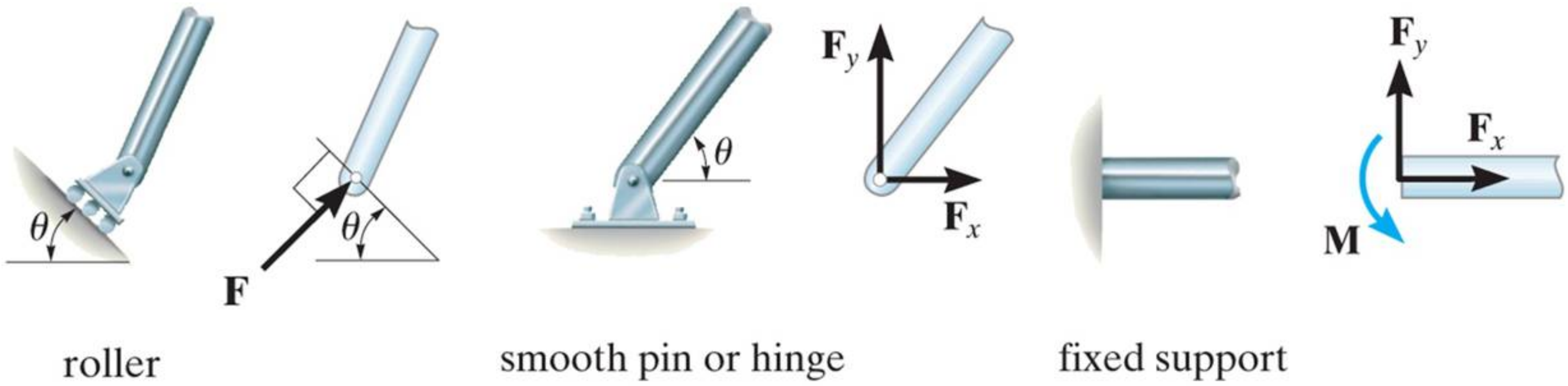
Idealized model



Free-body diagram

3. Label loads and dimensions on the FBD: All known forces and couple moments should be labeled with their magnitudes and directions. For the unknown forces and couple moments, use letters like A_x, A_y, M_A. Indicate any necessary dimensions.

SUPPORT REACTIONS IN 2-D



As a general rule, if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction.

Similarly, if rotation is prevented, a couple moment is exerted on the body in the opposite direction.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

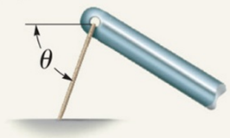
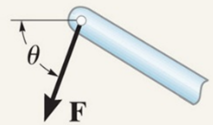
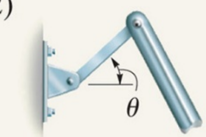
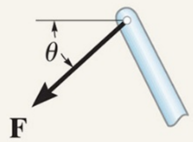
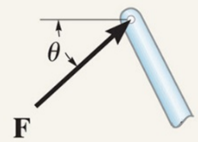
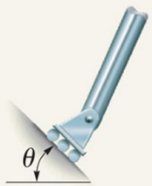
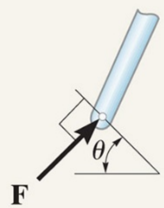
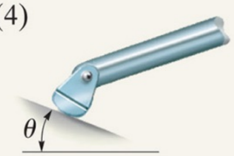
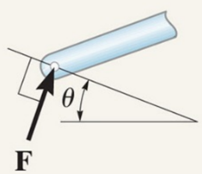
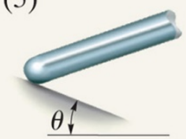
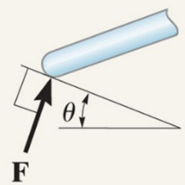
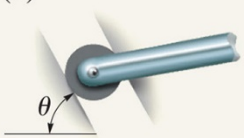
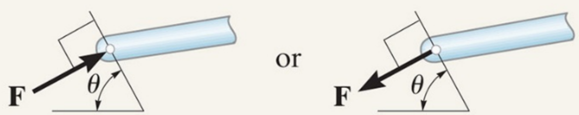
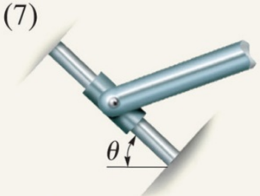
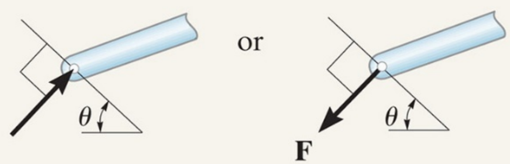
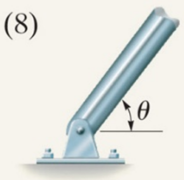
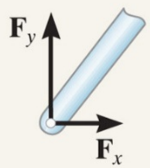
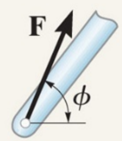
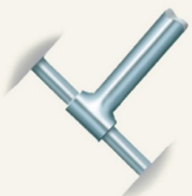
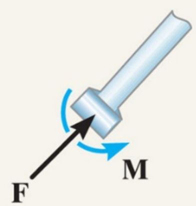
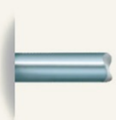
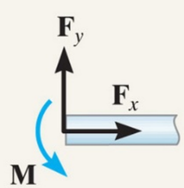
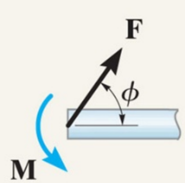
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

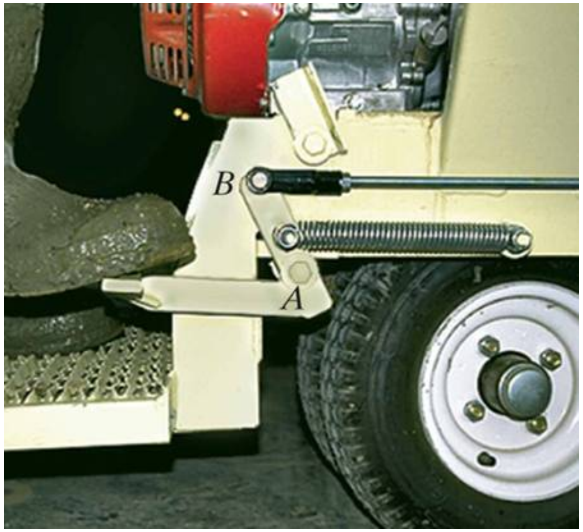
Types of Connection	Reaction	Number of Unknowns
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

continued

TABLE 5–1 Continued

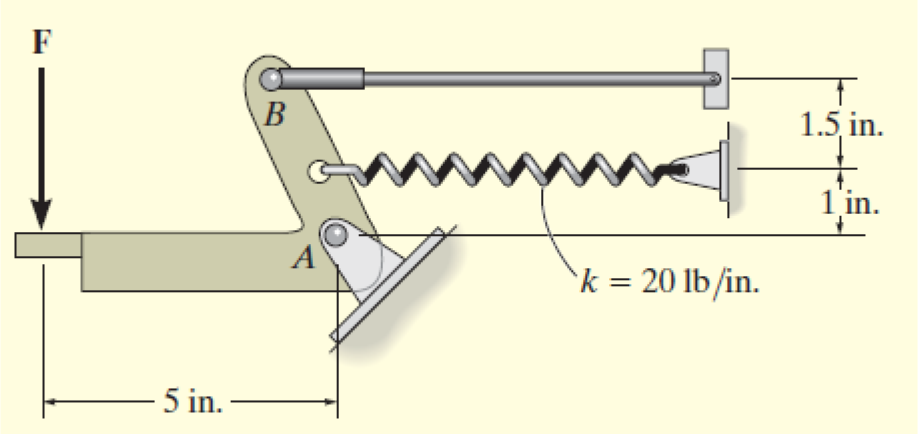
Types of Connection	Reaction			Number of Unknowns
(8)  smooth pin or hinge		or		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod				Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		or		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

EXAMPLE I

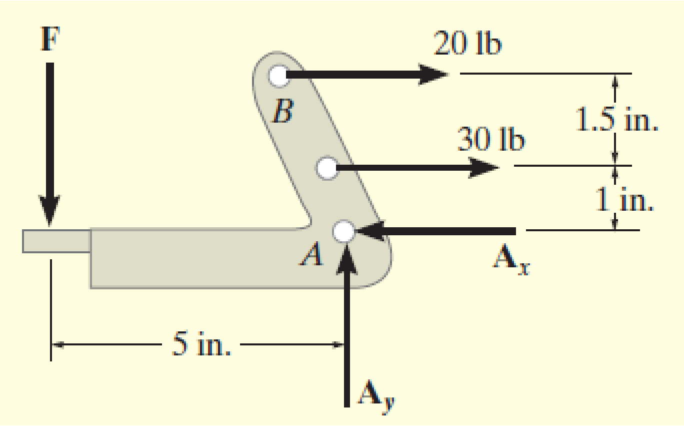


Given: The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B is 20 lb*.

Draw: An idealized model and free-body diagram of the foot pedal.



The idealized model



The free-body diagram

EXAMPLE II

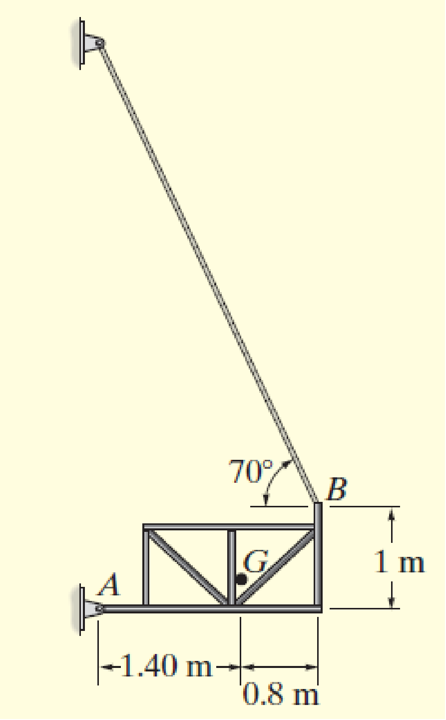
Given: The unloaded platform is suspended off the edge of the oil rig. The platform has a mass of 200 kg.

Draw: An idealized model and free-body diagram of the platform.

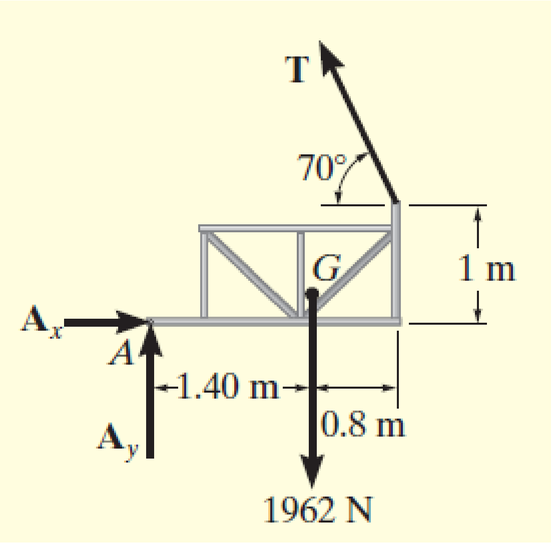
The idealized model of the platform is considered in two dimensions because the loading and the dimensions are all symmetrical about a vertical plane passing through its center.

EXAMPLE II (continued)

The connection at A is treated as a pin, and the cable supports the platform at B. Note the assumed directions of the forces! The point G is the center of gravity of the platform.



The idealized model



The free-body diagram

CONCEPT QUIZ

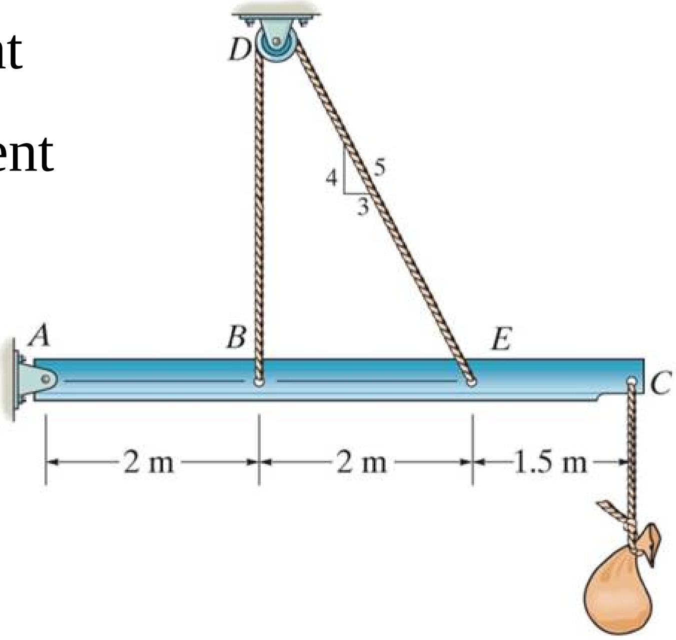
1. The beam and the cable (with a frictionless pulley at D) support an 80 kg load at C. In a FBD of only the beam, there are how many unknowns?

   A) Two forces and one couple moment

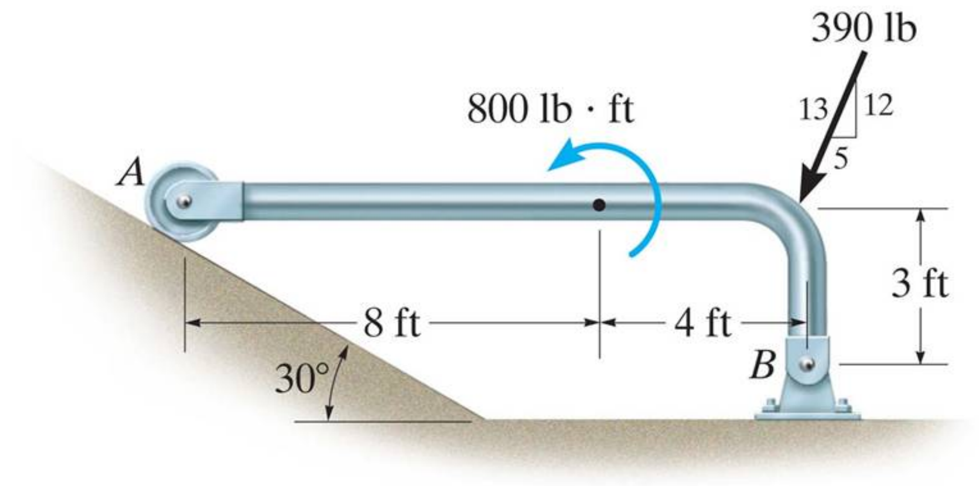
   B) Three forces and one couple moment

   C) Three forces

   D) Four forces



CONCEPT QUIZ (continued)

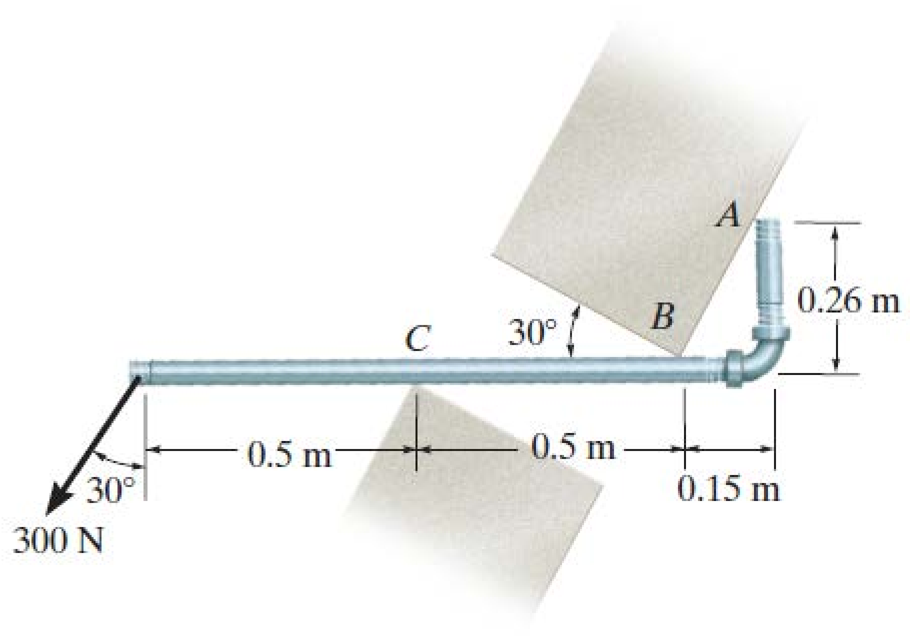


2. If the directions of the force and the couple moments are both reversed, what will happen to the beam?

   A) The beam will lift from A.
   B) The beam will lift at B.
   C) The beam will be restrained.
   D) The beam will break.

Example III

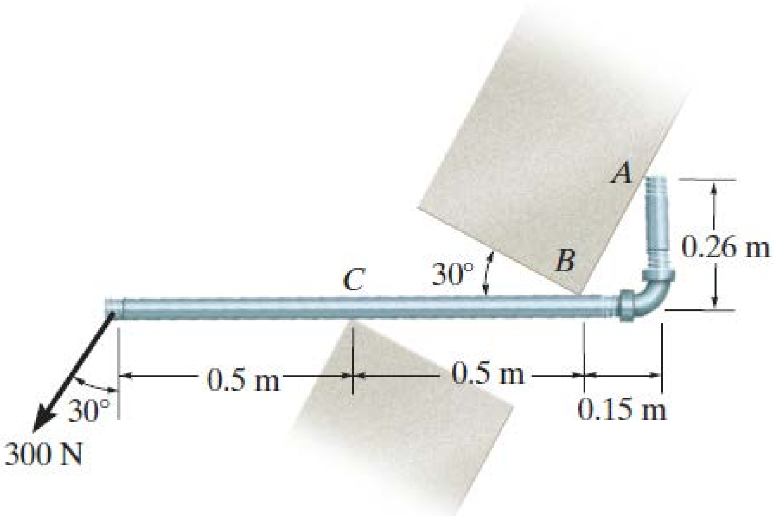
Given:



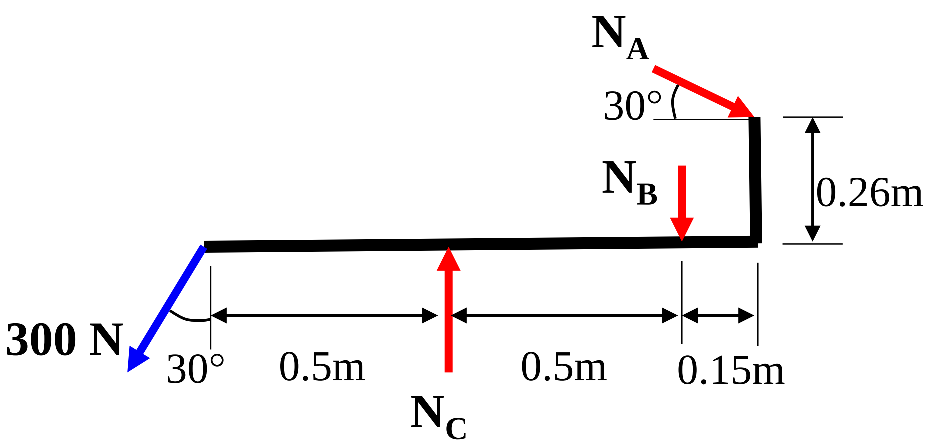
Draw:

A FBD of the smooth pipe which rests against the opening at the points of contact *A*, *B*, and *C*.

EXAMPLE III (continued)



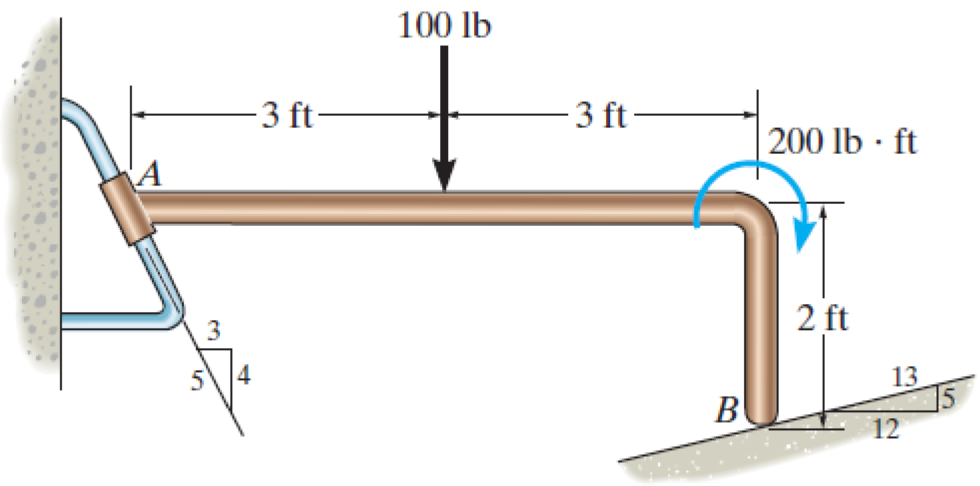
The idealized model



The free body diagram

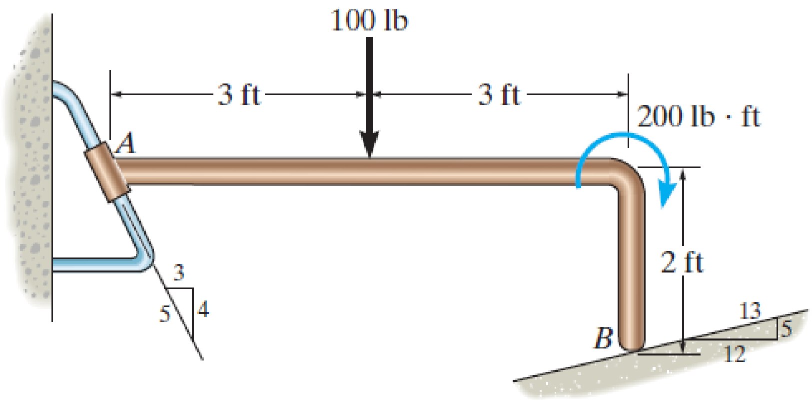
EXAMPLE IV

Given:

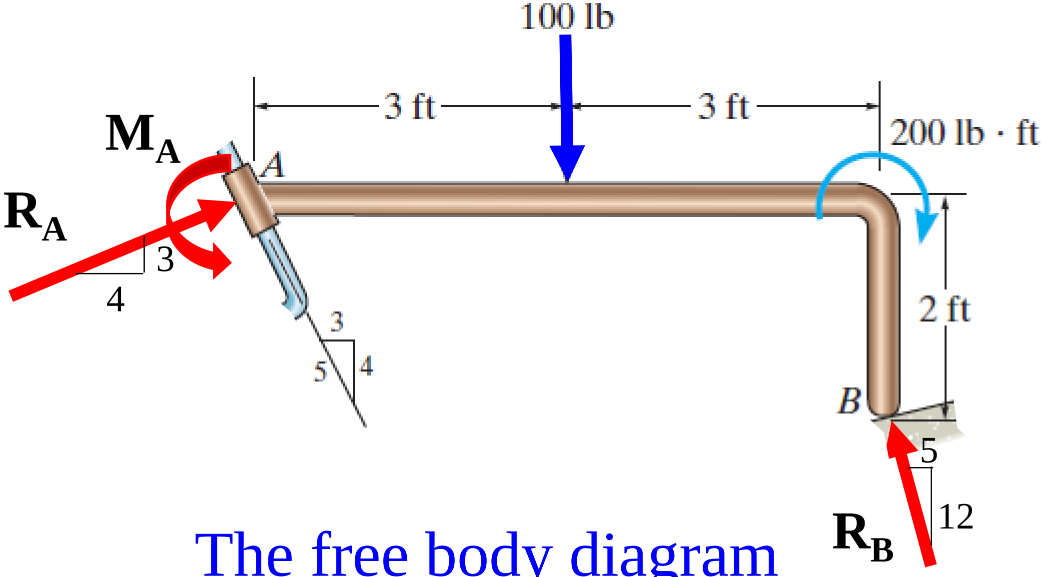


Draw: Draw a FBD of the bent rod supported by a smooth surface at *B* and by a collar at *A*, which is fixed to the rod and is free to slide over the fixed inclined rod.

EXAMPLE IV (continued)



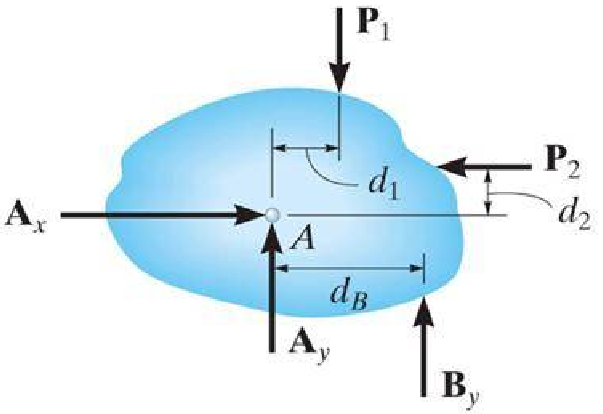
The idealized model



The free body diagram

EQUATIONS OF EQUILIBRIUM IN 2-D

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier). This 2-D condition can be represented by the three scalar equations:



$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

where point A is any arbitrary point.

EQUATIONS OF EQUILIBRIUM IN 2-D

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. There are two other sets of equilibrium equations that can be used.

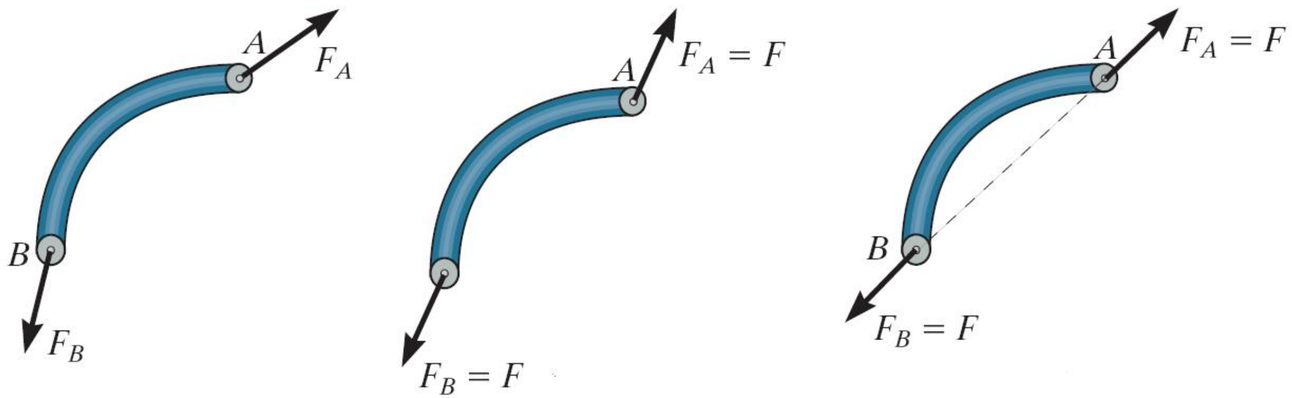
$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

$$\sum M_A = 0 \qquad \sum M_B = 0 \qquad \sum M_C = 0$$

where points A, B, and C are any arbitrary points.

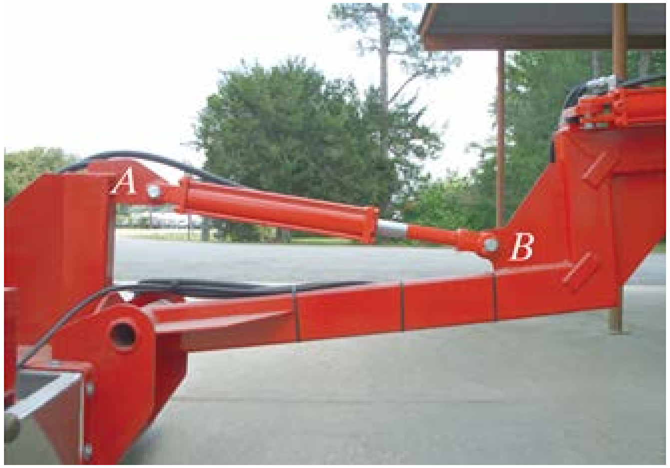
TWO-FORCE MEMBERS

The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B in the figure below).

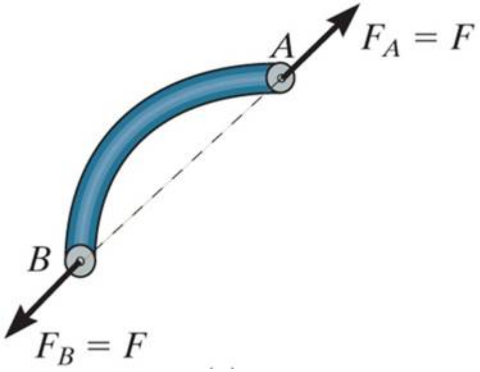


If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.

EXAMPLES OF TWO-FORCE MEMBERS

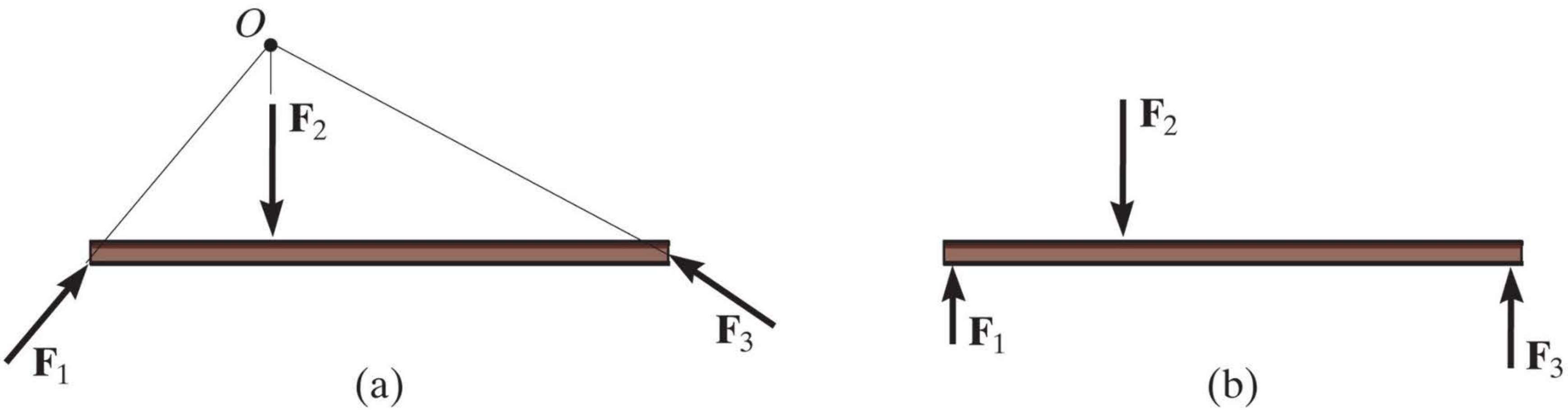


In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.



This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

THREE FORCE-MEMBERS



Three-force member

If a member is subjected to only three forces, it is called **three-force member**. These forces has to be **concurrent** or **parallel** (concurrent at infinity) to satisfy the moment equilibrium equation.

STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable x - y coordinate system.

2. Draw a free-body diagram (FBD) of the object under analysis.

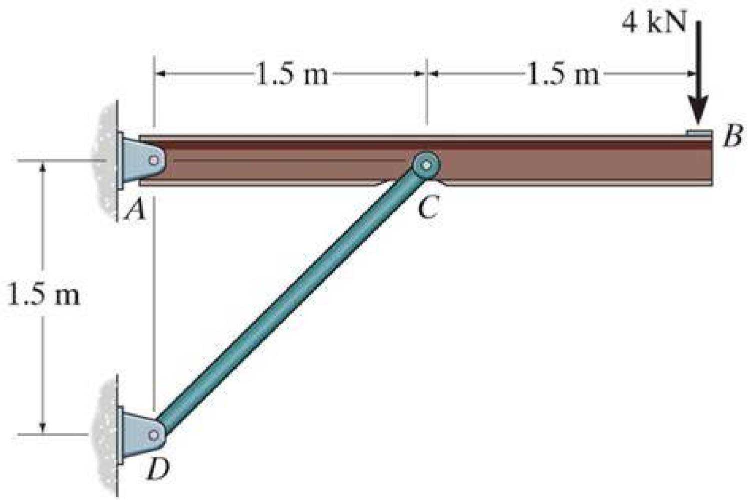
3. Apply the three equations of equilibrium (E-of-E) to solve for the unknowns.

IMPORTANT NOTES

1. If there are more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics. We will need the science of Mechanics of Deformable Solids (MODS) to solve such problems.

2. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem. It's better to consider all the unknown forces in the positive x and y directions, and the unknown moments in the positive z (CCW) direction.

EXAMPLE V



Given: The 4kN load at B of the beam is supported by pins at A and C.

Find: The support reactions at A and C.

Plan:

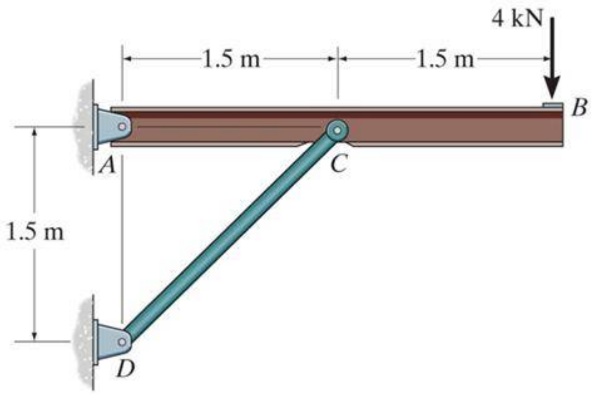
1. Put the x and y-axes in the horizontal and vertical directions, respectively.

2. Determine if there are any two-force members.

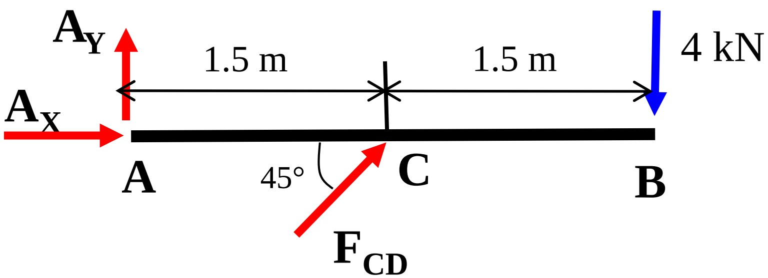
3. Draw a complete FBD of the boom.

4. Apply the E-of-E to solve for the unknowns.

EXAMPLE V (continued)



FBD of the beam:



Note: Upon recognizing CD as a two-force member, the number of unknowns at C is reduced from two to one. Now, using E-o-f E, we get,

$$\circlearrowleft + \sum M_A = F_{CD} \sin 45° \times 1.5 - 4 \times 3 = 0$$

$$F_{CD} = 11.31 \text{ kN} \quad \text{or} \quad \underline{11.3 \text{ kN}}$$

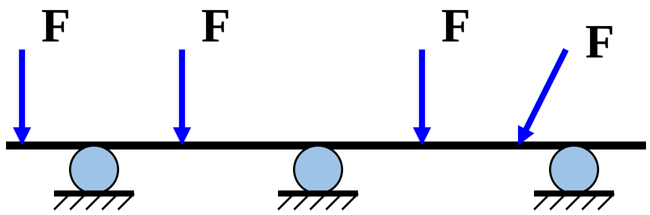
$$\rightarrow + \sum F_X = A_X + 11.31 \cos 45° = 0; \qquad \underline{A_X = -8.00 \text{ kN}}$$

$$\uparrow + \sum F_Y = A_Y + 11.31 \sin 45° - 4 = 0; \qquad \underline{A_Y = -4.00 \text{ kN}}$$

Note that the negative signs means that the reactions have the opposite directions to that assumed (as originally shown on FBD).

CONCEPT QUIZ

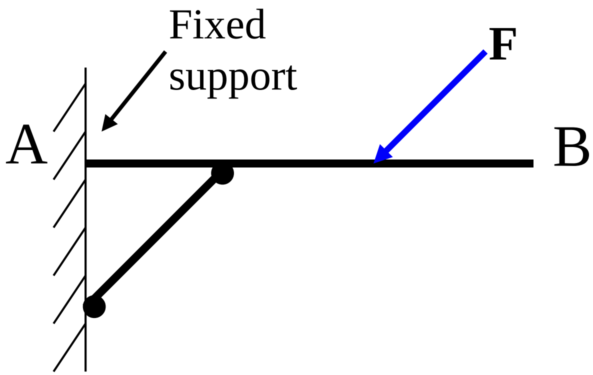
1. For this beam, how many support reactions are there, and is the problem statically determinate?



A) (2, Yes) B) (2, No)

C) (3, Yes) D) (3, No)

2. The beam AB is loaded and supported as shown: a) how many support reactions are there on the beam, b) is this problem statically determinate, and c) is the structure stable?

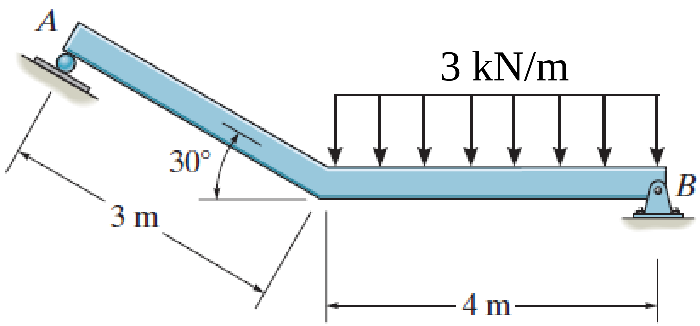


A) (4, Yes, No) B) (4, No, Yes)

C) (5, Yes, No) D) (5, No, Yes)

PEARSON

EXAMPLE VI



Given: The beam is supported by the roller at A and a pin at B.

Find: The reactions at points A and B on the beam.

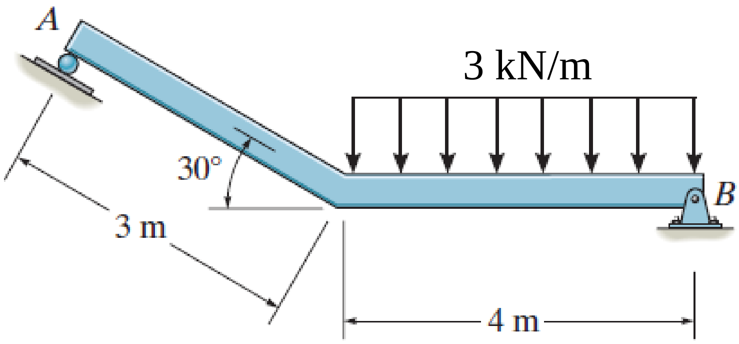
Plan:

a) Establish the x–y axis system.

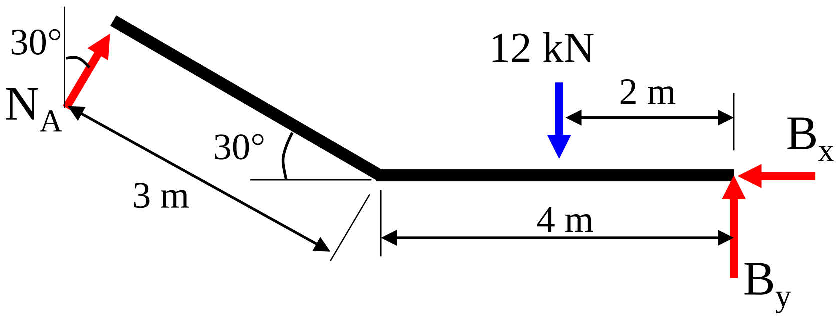
b) Draw a complete FBD of the beam.

c) Apply the E-of-E to solve for the unknowns.

EXAMPLE VI (continued)



FBD of the beam



Note that the distributed load has been reduced to a single force.

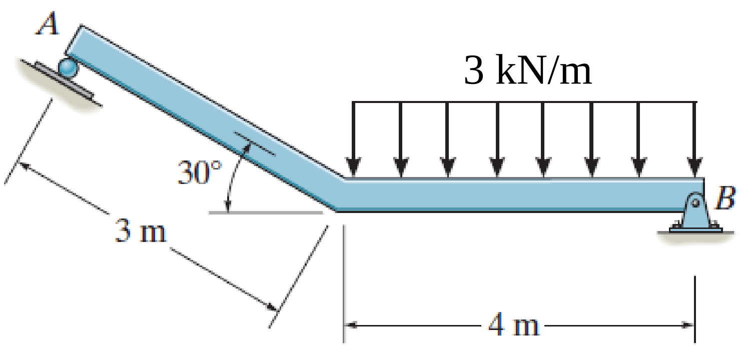
First, write a moment equation about point B. Why point B?

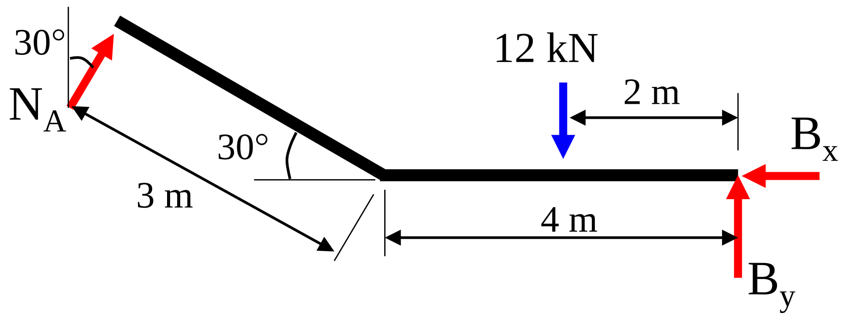
$$\circlearrowleft + \sum M_B = -(N_A \cos 30°) \times (4 + 3 \cos 30°) - (N_A \sin 30°) \times (3 \sin 30°) + 12 \times 2 = 0$$

N_A = 3.713 = 3.71 kN

EXAMPLE VI (continued)

FBD of the beam





Recall N_A = 3.713 =3.71 kN

Now write the $\sum F_X = \sum F_Y = 0$ equations.

$\rightarrow + \sum F_X = 3.713 \sin 30° - B_x = 0$

$\uparrow + \sum F_Y = 3.713 \cos 30° - 12 + B_y = 0$

Solving these two equations, we get

$B_x = 1.86$ kN $\leftarrow$

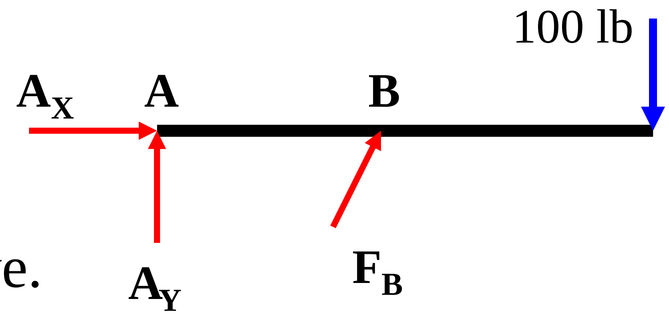
$B_y = 8.78$ kN $\uparrow$

QUIZ

1. Which equation of equilibrium allows you to determine F_B right away?

   A) $\sum F_X = 0$ B) $\sum F_Y = 0$

   C) $\sum M_A = 0$ D) Any one of the above.



2. A beam is supported by a pin joint and a roller. How many support reactions are there and is the structure stable for all types of loadings?

   A) (3, Yes) B) (3, No)

   C) (4, Yes) D) (4, No)