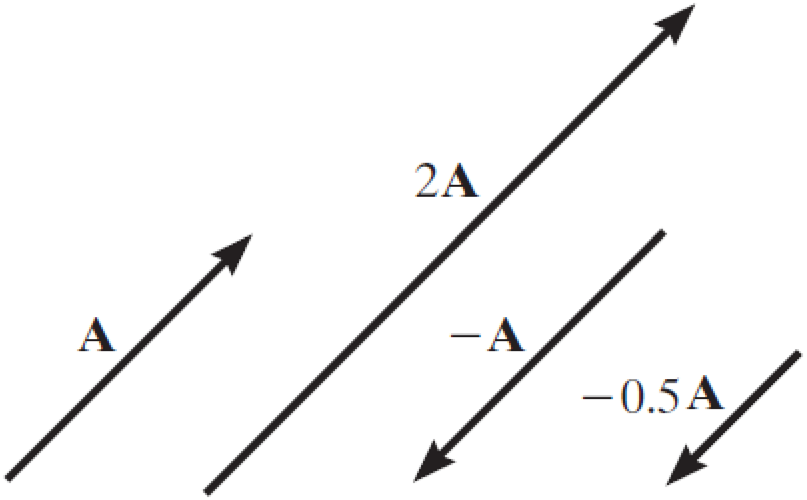
SCALARS AND VECTORS
(Section 2.1)

	Scalars	***Vectors***
Examples:	Mass, Volume	Force, Velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a "carrot"

In these PowerPoint presentations, a vector quantity is represented ***like this*** (in **bold**, *italics*, and red).

VECTOR OPERATIONS (Section 2.2)

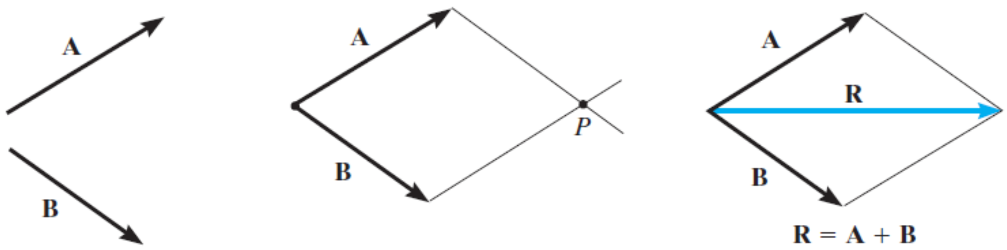


Scalar Multiplication

and Division

VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

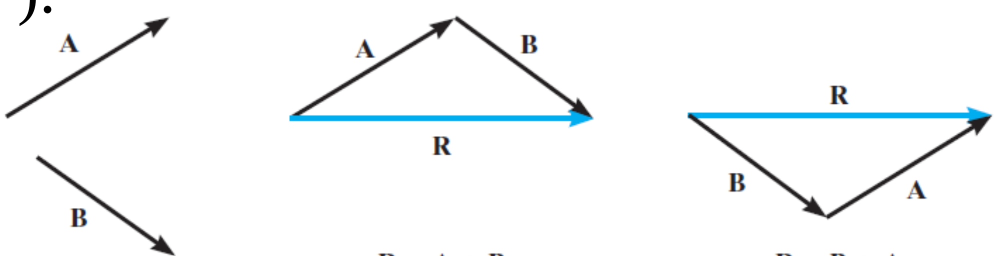
Parallelogram Law:



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Parallelogram law

Triangle method
(always 'tip to tail'):



$\mathbf{R} = \mathbf{A} + \mathbf{B}$
Triangle rule

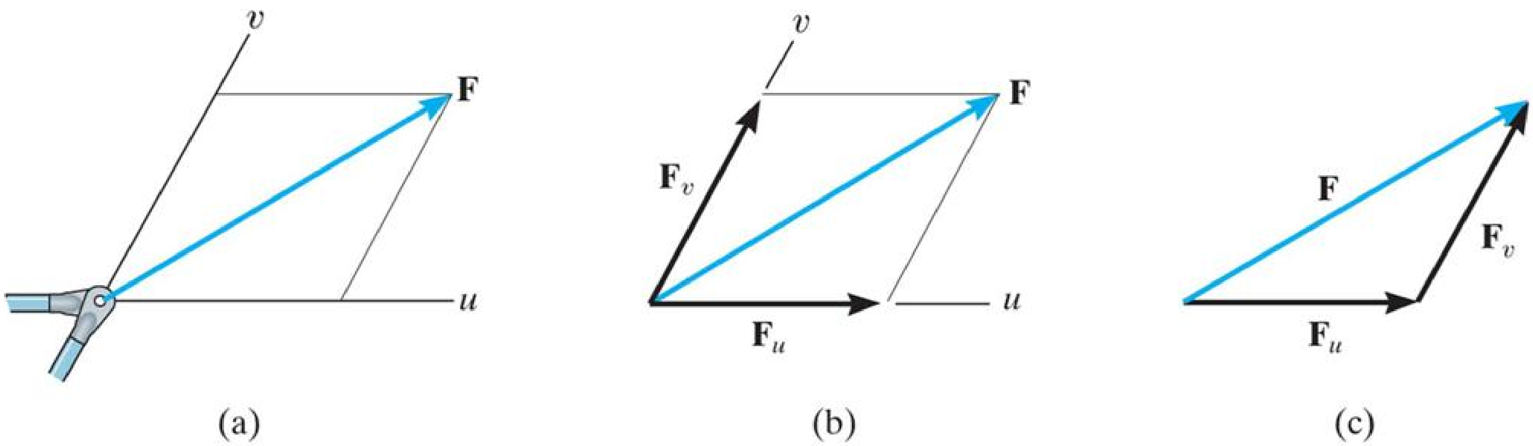
$\mathbf{R} = \mathbf{B} + \mathbf{A}$
Triangle rule

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?

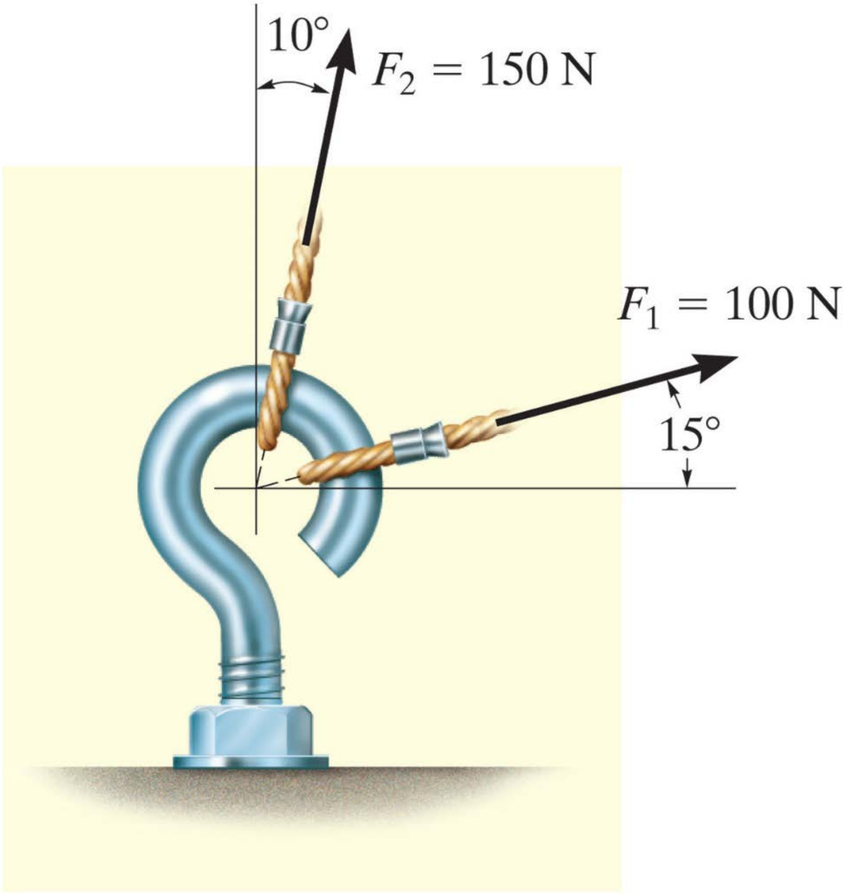
RESOLUTION OF A VECTOR

"Resolution" of a vector is breaking up a vector into components.



It is kind of like using the parallelogram law in reverse.

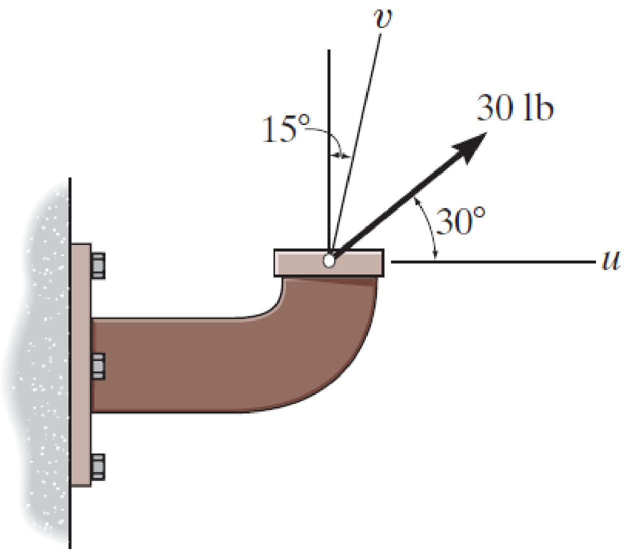
EXAMPLE I



Given: Forces F_1 and F_2

Find: The magnitude and direction of the resultant force

EXAMPLE II



Given: A force acting on a pipe.

Find: Resolve the force into components along the u and v-axes, and determine the magnitude of each of these components.

Plan:

a) Construct lines parallel to the u and v-axes, and form a parallelogram.

b) Resolve the forces into their u-v components.

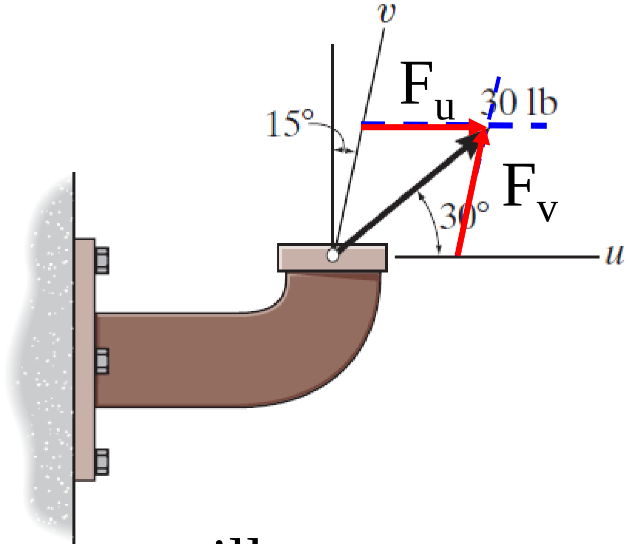
c) Find magnitude of the components from the law of sines.

EXAMPLE II (continued)

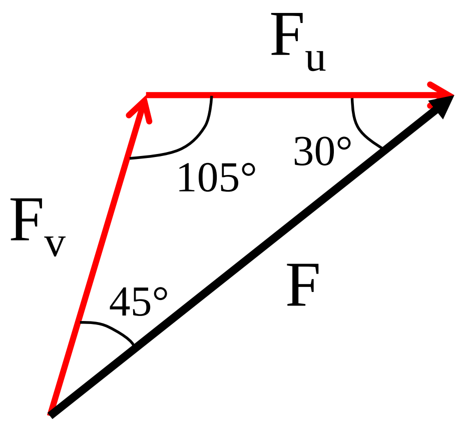
Solution:

Draw lines parallel to the u and v-axes.

And resolve the forces into the u-v components.

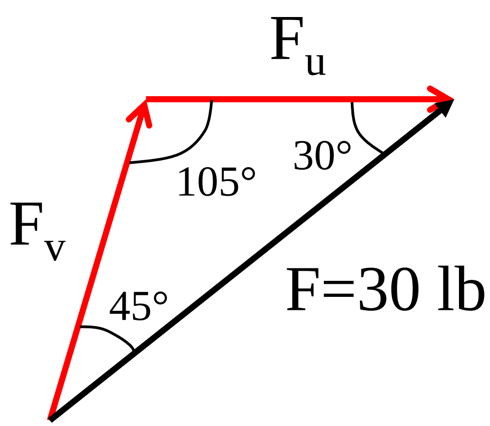


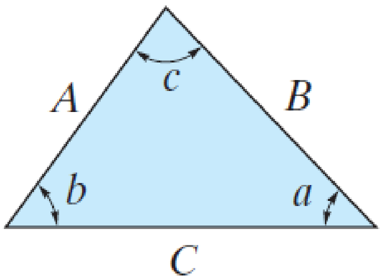
Redraw the top portion of the parallelogram to illustrate a Triangular, head-to-tail, addition of the components.



EXAMPLE II (continued)

The magnitudes of two force components are determined from the law of sines.





Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

$$\frac{30}{\sin 105^\circ} = \frac{F_u}{\sin 45^\circ} = \frac{F_v}{\sin 30^\circ}$$

F_u = (30/sin105°) sin 45° = 22.0 lb

F_v = (30/sin105°) sin 30° = 15.5 lb

QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?

   A) Yes, but not uniquely.

   B) No.

   C) Yes, uniquely.

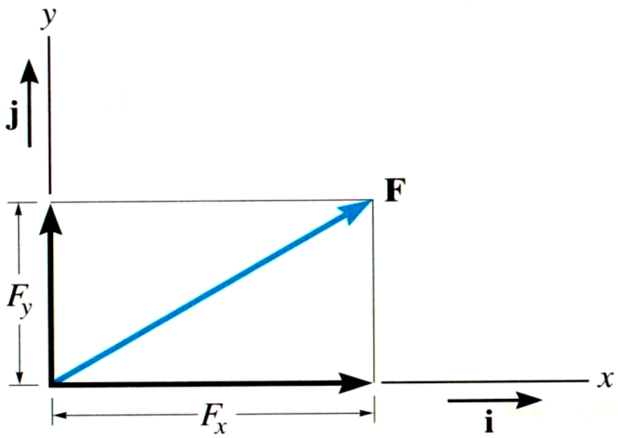
2. Can you resolve a 2-D vector along three directions (say at 0, 60, and 120°)?

   A) Yes, but not uniquely.

   B) No.

   C) Yes, uniquely.

ADDITION OF A SYSTEM OF COPLANAR FORCES (Section 2.4)



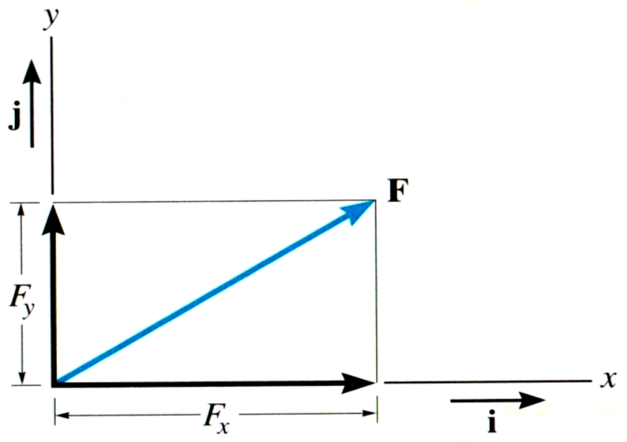
(a)

- We ‘resolve’ vectors into components using the x and y-axis coordinate system.
- Each component of the vector is shown as a magnitude and a direction.

- The directions are based on the x and y axes. We use the “unit vectors” ***i*** and ***j*** to designate the x and y-axes.

For example,

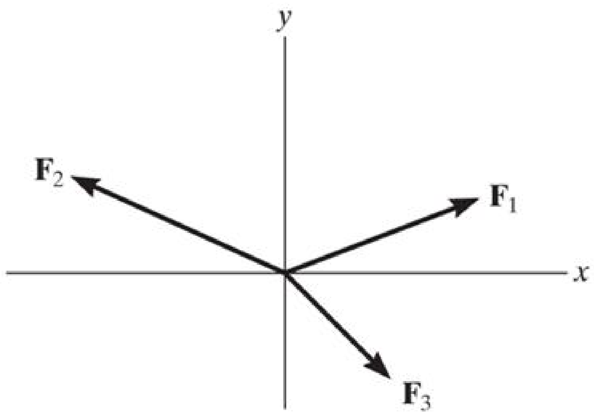
$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$$

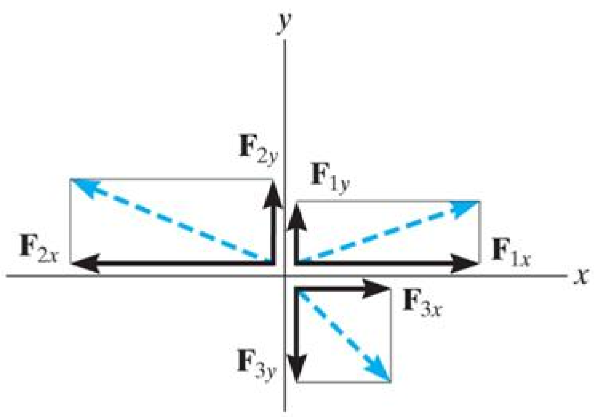


(a)

The x and y-axis are always perpendicular to each other.

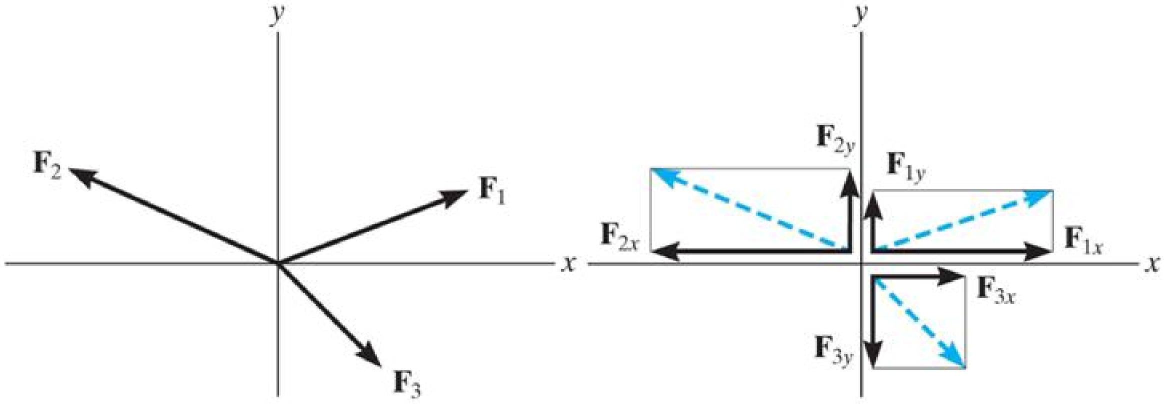
ADDITION OF SEVERAL VECTORS





- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

An example of the process:



Break the three vectors into components, then add them.

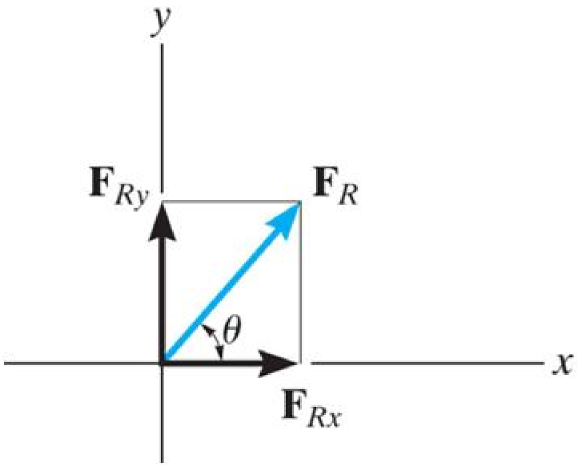
$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$

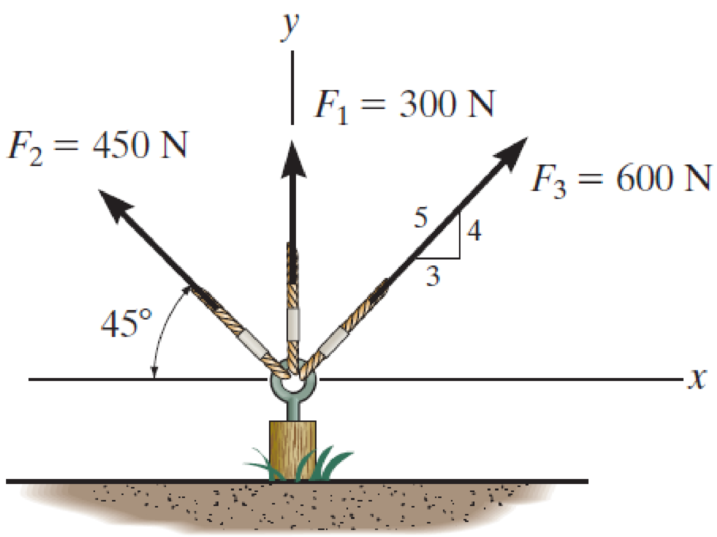
You can also represent a 2-D vector with a magnitude and angle.



$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

EXAMPLE III



Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

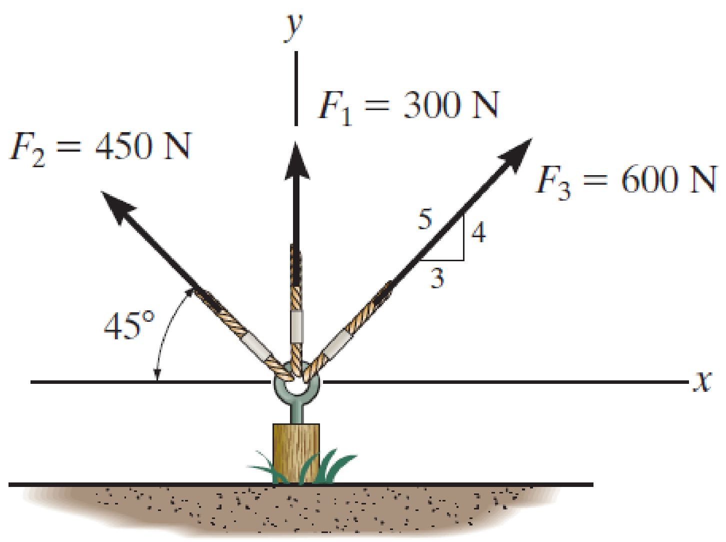
Plan:

a) Resolve the forces into their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE III (continued)



$\boldsymbol{F_1} = \{0\ \boldsymbol{i} + 300\ \boldsymbol{j}\}$ N

$\boldsymbol{F_2} = \{-\ 450 \cos(45°)\ \boldsymbol{i} + 450 \sin(45°)\ \boldsymbol{j}\}$ N

$= \{-\ 318.2\ \boldsymbol{i} + 318.2\ \boldsymbol{j}\}$ N

$\boldsymbol{F_3} = \{(3/5)\ 600\ \boldsymbol{i} + (4/5)\ 600\ \boldsymbol{j}\}$ N

$= \{360\ \boldsymbol{i} + 480\ \boldsymbol{j}\}$ N

EXAMPLE III (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

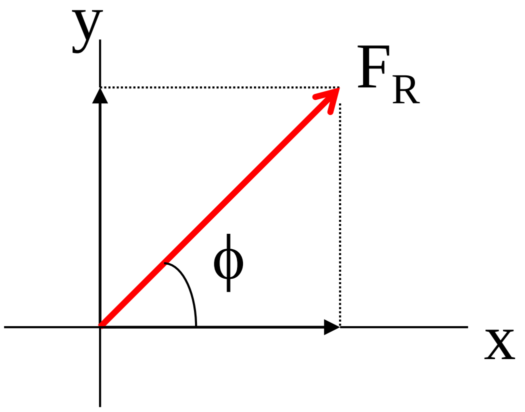
$\boldsymbol{F_R}$ = { (0 – 318.2 + 360) ***i*** + (300 + 318.2 + 480) ***j*** } N

= { 41.80 ***i*** + 1098 ***j*** } N

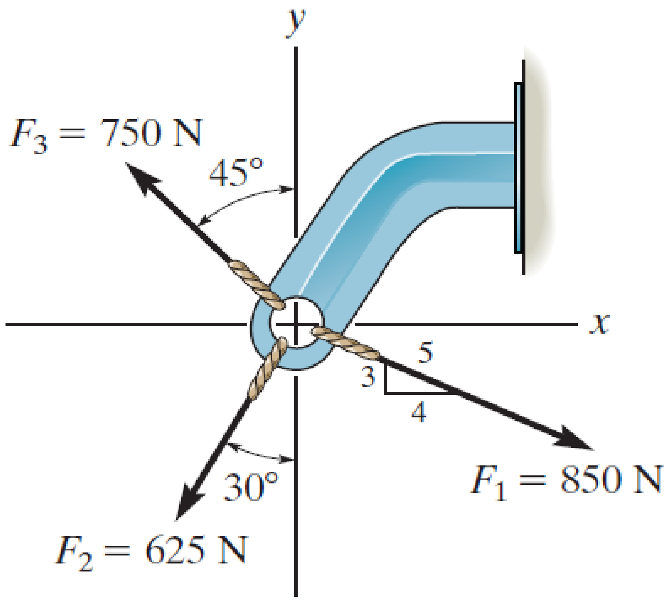
Using magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2}$ = <u>1099 N</u>

$\phi = \tan^{-1}(1098/41.80)$ = <u>87.8°</u>



Example IV



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force. Show the resultant in a sketch.

Plan:

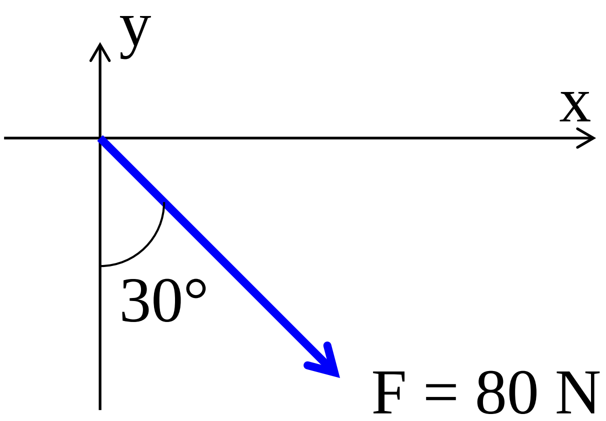
a) Resolve the forces into their x and y-components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

QUIZ

1. Resolve ***F*** along x and y axes and write it in vector form. ***F*** = { ____________ } N



A) 80 cos (30°) ***i*** – 80 sin (30°) ***j***

B) 80 sin (30°) ***i*** + 80 cos (30°) ***j***

C) 80 sin (30°) ***i*** – 80 cos (30°) ***j***

D) 80 cos (30°) ***i*** + 80 sin (30°) ***j***

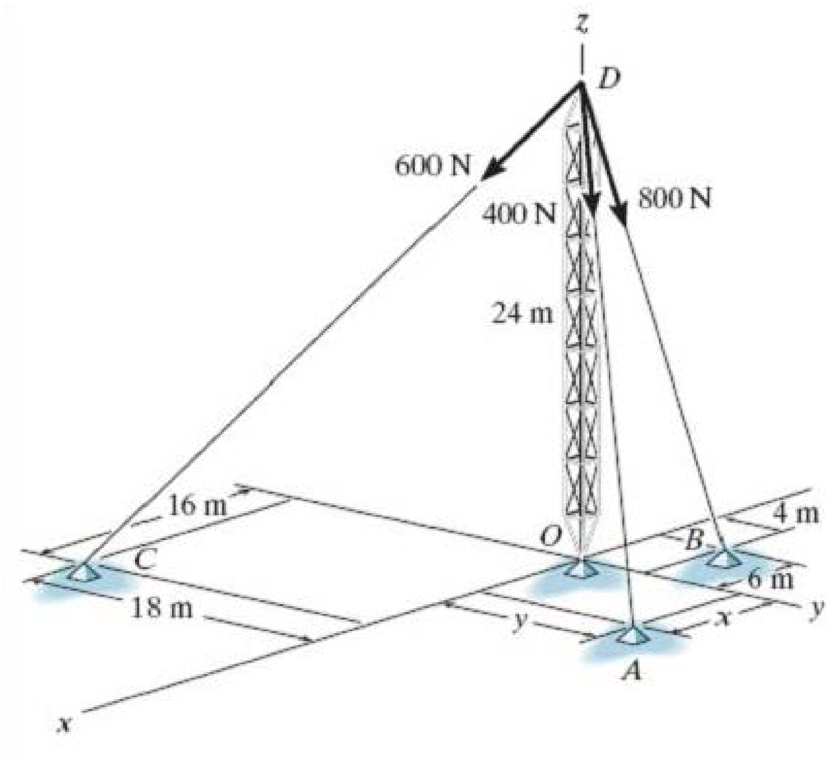
2. Determine the magnitude of the resultant ($\boldsymbol{F_1}$ + $\boldsymbol{F_2}$) force in N when $\boldsymbol{F_1}$ = { 10 ***i*** + 20 ***j*** } N and $\boldsymbol{F_2}$ = { 20 ***i*** + 20 ***j*** } N .

A) 30 N B) 40 N C) 50 N

D) 60 N E) 70 N

Cartesian Vectors (3D)
APPLICATIONS (continued)

In the case of this radio tower, if you know the forces in the three cables, how would you determine the resultant force acting at D, the top of the tower?



CARTESIAN UNIT VECTORS

For a vector ***A***, with a magnitude of A, an unit vector is defined as

$$\boldsymbol{u}_A = \boldsymbol{A} \,/\, A\,.$$

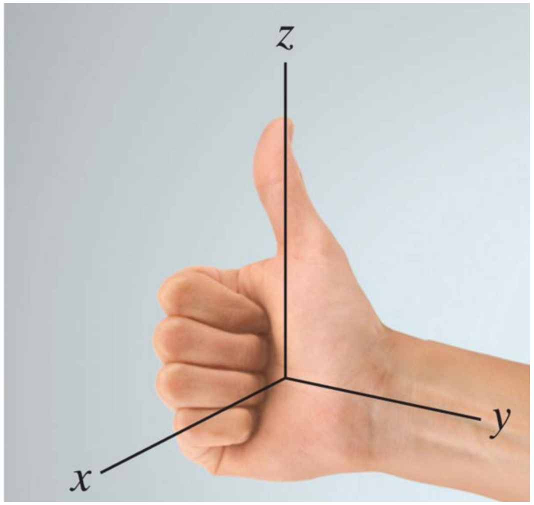
Characteristics of a unit vector **:**

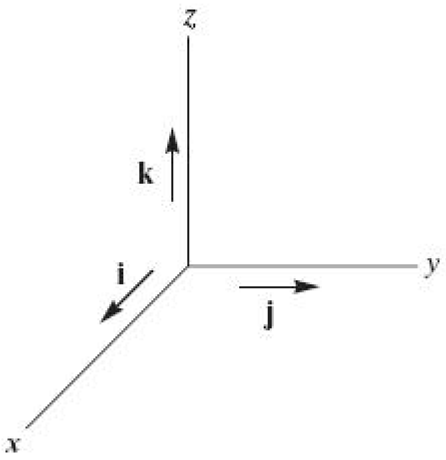
a) Its magnitude is 1.

b) It is dimensionless (has no units).

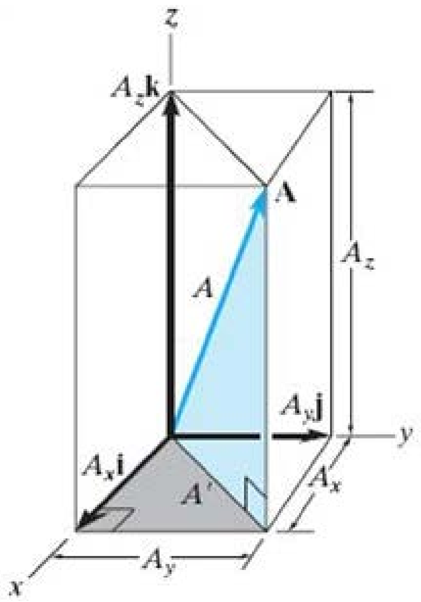
c) It points in the same direction as the original vector (***A***).

The unit vectors in the Cartesian axis system are ***i***, ***j***, and ***k***. They are unit vectors along the positive x, y, and z axes respectively.





CARTESIAN VECTOR REPRESENTATION



Consider a box with sides A_X, A_Y, and A_Z meters long.

The vector ***A*** can be defined as

$$\boldsymbol{A} = (A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k})\ \text{m}$$

The projection of vector ***A*** in the x-y plane is A´. The magnitude of A´ is found by using the same approach as a 2-D vector: $A' = (A_X^2 + A_Y^2)^{1/2}$.

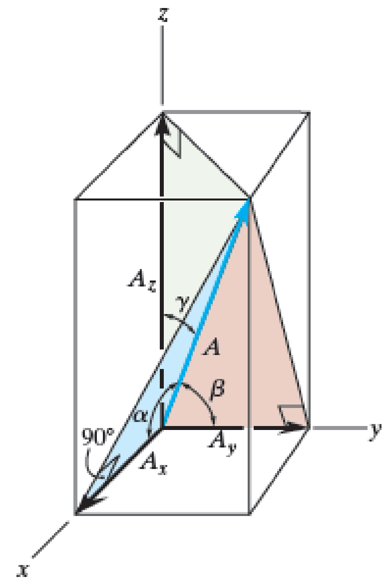
The magnitude of the position vector ***A*** can now be obtained as

$$A = ((A')^2 + A_Z^2)^{1/2} = (A_X^2 + A_Y^2 + A_Z^2)^{1/2}$$

DIRECTION OF A CARTESIAN VECTOR

The direction or orientation of vector **A** is defined by the angles α, β, and γ.

These angles are measured between the vector and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°



Using trigonometry, "direction cosines" are found using

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}}$$

or written another way, $\boldsymbol{u}_A = \cos\alpha\,\boldsymbol{i} + \cos\beta\,\boldsymbol{j} + \cos\gamma\,\boldsymbol{k}$.

ADDITION OF CARTESIAN VECTORS (Section 2.6)

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

$$\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}}$$

For example, if

$\boldsymbol{A} = A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}$ and

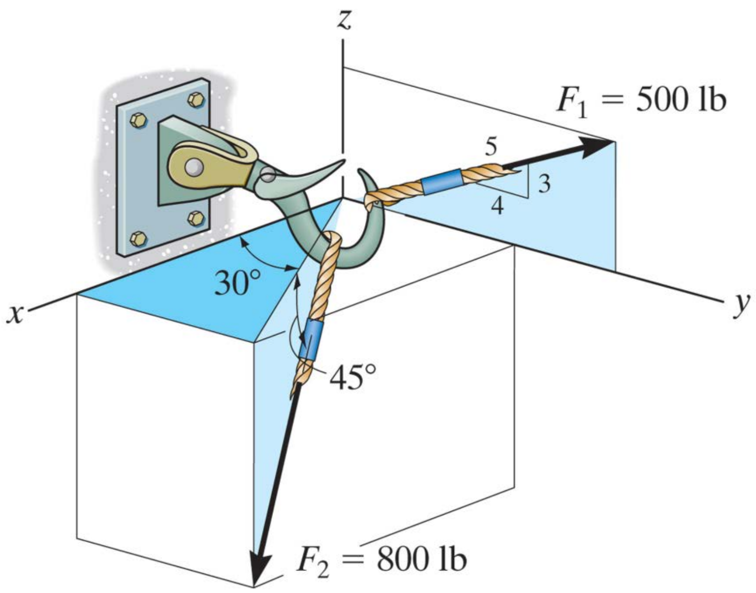
$\boldsymbol{B} = B_X\,\boldsymbol{i} + B_Y\,\boldsymbol{j} + B_Z\,\boldsymbol{k}$, then

$$\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\,\boldsymbol{i} + (A_Y + B_Y)\,\boldsymbol{j} + (A_Z + B_Z)\,\boldsymbol{k}$$

or

$$\boldsymbol{A} - \boldsymbol{B} = (A_X - B_X)\,\boldsymbol{i} + (A_Y - B_Y)\,\boldsymbol{j} + (A_Z - B_Z)\,\boldsymbol{k}\,.$$

EXAMPLE V



Given: Two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are applied to a hook.

Find: The resultant force in Cartesian vector form.

Plan:

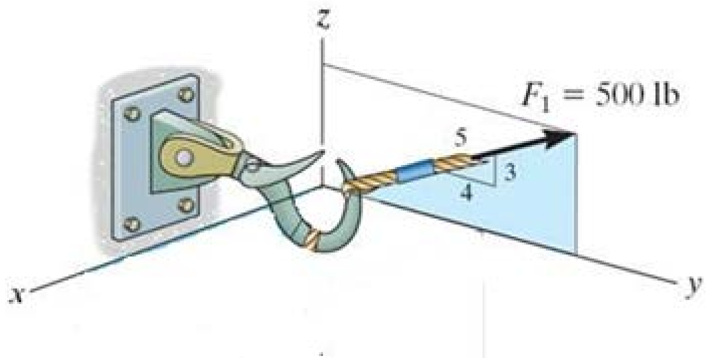
1) Using geometry and trigonometry, write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in Cartesian vector form.

2) Then add the two forces (by adding x and y-components).

EXAMPLE V (continued)

Solution:

First, resolve force $\boldsymbol{F_1}$.



$F_x = 0 = 0$ lb

$F_y = 500\ (4/5) = 400$ lb

$F_z = 500\ (3/5) = 300$ lb

Now, write $\boldsymbol{F_1}$ in Cartesian vector form (don't forget the units!).

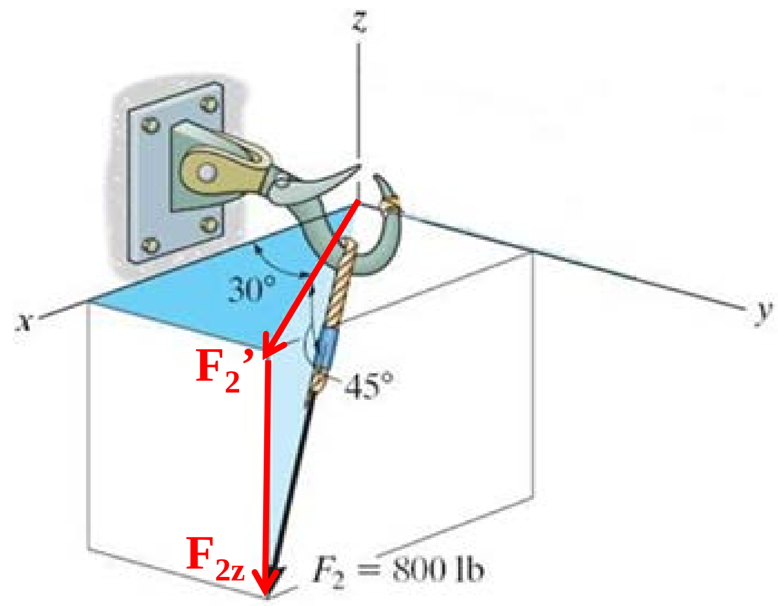
$\boldsymbol{F_1} = \{0\ \boldsymbol{i} + 400\ \boldsymbol{j} + 300\ \boldsymbol{k}\}$ lb

EXAMPLE V (continued)

Now, resolve force ***F₂***.

F_{2z} = -800 sin 45° = – 565.7 lb

F_2’ = 800 cos 45° = 565.7 lb



F_2’ can be further resolved as,

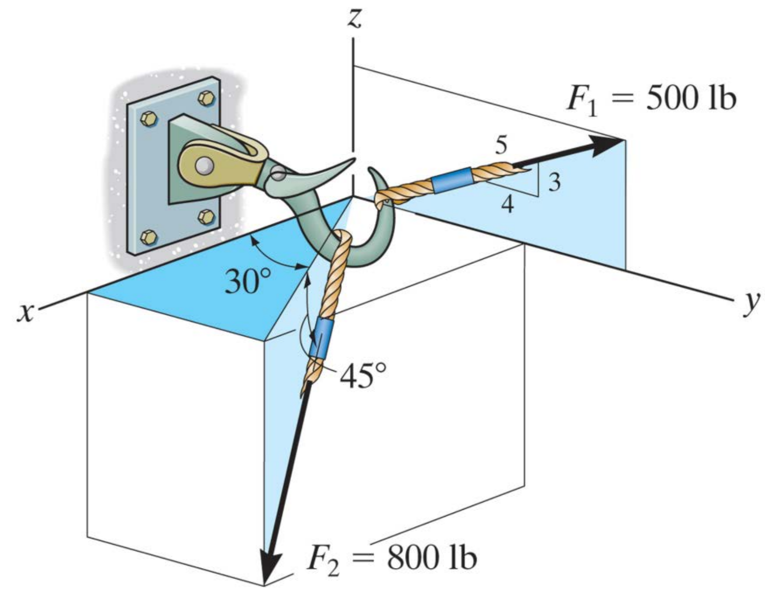
F_{2x} = 565.7 cos 30° = 489.9 lb

F_{2y} = 565.7 sin 30° = 282.8 lb

Thus, we can write:

$\boldsymbol{F_2}$ = {489.9 ***i*** + 282.8 ***j*** – 565.7 ***k*** } lb

EXAMPLE V (continued)



So $\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$ and

$\boldsymbol{F_1} = \{0\,\boldsymbol{i} + 400\,\boldsymbol{j} + 300\,\boldsymbol{k}\}$ lb

$\boldsymbol{F_2} = \{489.9\,\boldsymbol{i} + 282.8\,\boldsymbol{j} - 565.7\,\boldsymbol{k}\}$ lb

$\boldsymbol{F_R} = \{\underline{490\,\boldsymbol{i}} + \underline{683\,\boldsymbol{j} - 266\,\boldsymbol{k}}\}$ $\underline{\text{lb}}$

PEARSON

CONCEPT QUIZ

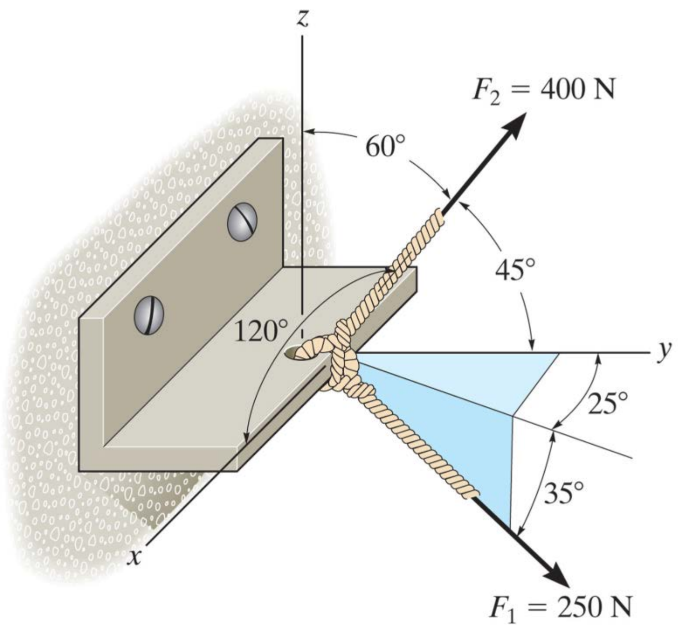
1. If you know only u_A, you can determine the ________ of ***A*** uniquely.

   A) magnitude
   B) angles (α, β and γ)
   C) components (A_X, A_Y, & A_Z)
   D) All of the above.

2. For a force vector, the following parameters are randomly generated. The magnitude is 0.9 N, $\alpha = 30^\circ$, $\beta = 70^\circ$, $\gamma = 100^\circ$. What is wrong with this 3-D vector ?

   A) Magnitude is too small.
   B) Angles are too large.
   C) All three angles are arbitrarily picked.
   D) All three angles are between 0° to 180°.

EXAMPLE VI



Given: The screw eye is subjected to two forces, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$.

Find: The magnitude and the coordinate direction angles of the resultant force.

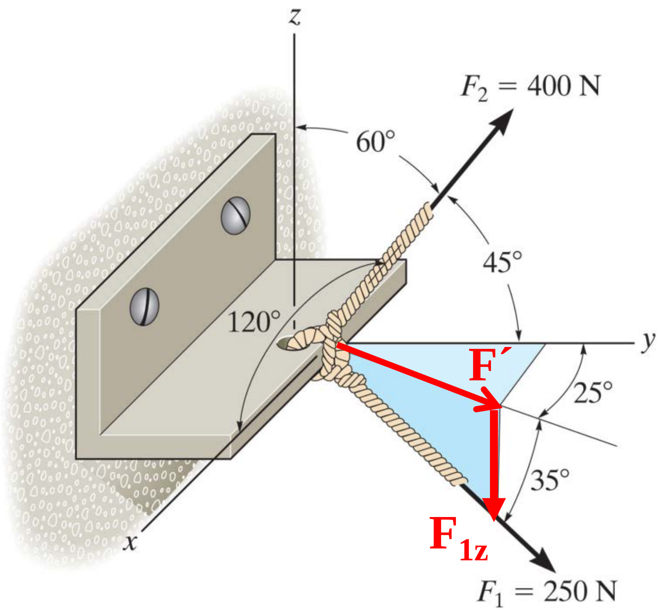
Plan:

1) Using the geometry and trigonometry, resolve and write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the Cartesian vector form.

2) Add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to get $\boldsymbol{F_R}$.

3) Determine the magnitude and angles α, β, γ.

EXAMPLE VI (continued)



First resolve the force $\boldsymbol{F_1}$.

$F_{1z} = -250 \sin 35° = -143.4$ N

$F´ = 250 \cos 35° = 204.8$ N

F´ can be further resolved as,

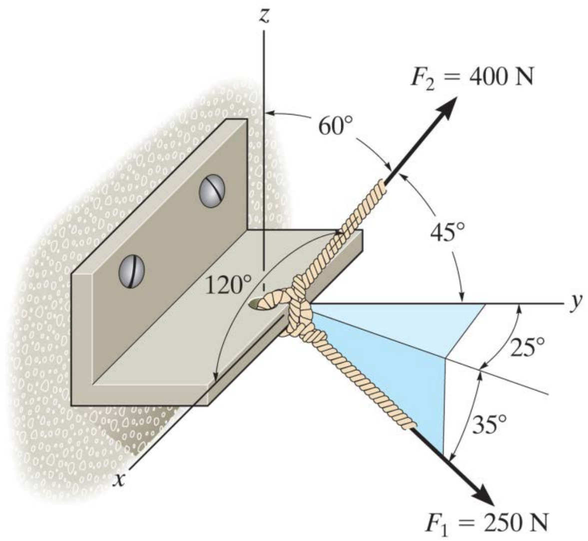
$F_{1x} = 204.8 \sin 25° = 86.6$ N

$F_{1y} = 204.8 \cos 25° = 185.6$ N

Now we can write:

$\boldsymbol{F_1} = \{86.6\, \boldsymbol{i} + 185.6\, \boldsymbol{j} - 143.4\, \boldsymbol{k}\}$ N

EXAMPLE VI (continued)



Now, resolve force $\boldsymbol{F_2}$.

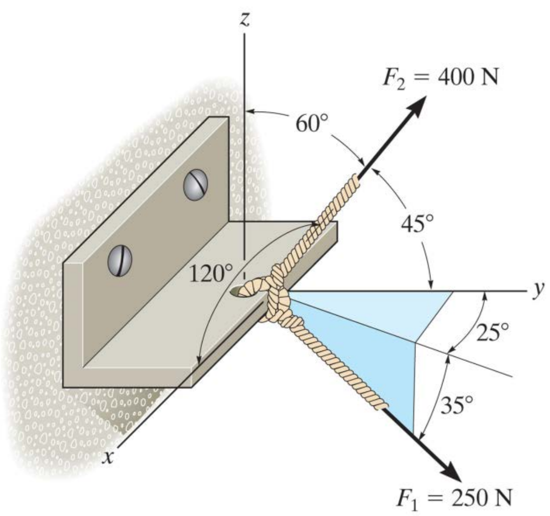
The force $\boldsymbol{F_2}$ can be represented in the Cartesian vector form as:

$$\boldsymbol{F_2} = 400\{ \cos 120° \, \boldsymbol{i} + \cos 45° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k} \} \text{ N}$$

$$= \{ -200 \, \boldsymbol{i} + 282.8 \, \boldsymbol{j} + 200 \, \boldsymbol{k} \} \text{ N}$$

$$\boldsymbol{F_2} = \{ -200 \, \boldsymbol{i} + 282.8 \, \boldsymbol{j} + 200 \, \boldsymbol{k} \} \text{ N}$$

EXAMPLE VI (continued)



So $\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$ and

$\boldsymbol{F_1}$ = { 86.6 $\boldsymbol{i}$ + 185.6 $\boldsymbol{j}$ – 143.4 $\boldsymbol{k}$} N

$\boldsymbol{F_2}$ = { -200 $\boldsymbol{i}$ + 282.8 $\boldsymbol{j}$ + 200 $\boldsymbol{k}$} N

$\boldsymbol{F_R}$ = { -113.4 $\boldsymbol{i}$ + 468.4 $\boldsymbol{j}$ + 56.6 $\boldsymbol{k}$} N

Now find the magnitude and direction angles for the vector.

$F_R = \{(-113.4)^2 + 468.4^2 + 56.6^2\}^{1/2} = 485.2 = \underline{485\text{ N}}$

$\alpha = \cos^{-1} (F_{Rx} / F_R) = \cos^{-1} (-113.4 / 485.2) = \underline{104°}$

$\beta = \cos^{-1} (F_{Ry} / F_R) = \cos^{-1} (468.4 / 485.2) = \underline{15.1°}$

$\gamma = \cos^{-1} (F_{Rz} / F_R) = \cos^{-1} (56.6 / 485.2) = \underline{83.3°}$

QUIZ

1. What is not true about an unit vector, e.g., $\boldsymbol{u_A}$?

   A) It is dimensionless.

   B) Its magnitude is one.

   C) It always points in the direction of positive X- axis.

   D) It always points in the direction of vector $\boldsymbol{A}$.

2. If $\boldsymbol{F} = \{10\,\boldsymbol{i} + 10\,\boldsymbol{j} + 10\,\boldsymbol{k}\}$ N and

   $\boldsymbol{G} = \{20\,\boldsymbol{i} + 20\,\boldsymbol{j} + 20\,\boldsymbol{k}\}$ N, then $\boldsymbol{F} + \boldsymbol{G} = \{$ _____ $\}$ **N**

   A) $10\,\boldsymbol{i} + 10\,\boldsymbol{j} + 10\,\boldsymbol{k}$

   B) $30\,\boldsymbol{i} + 20\,\boldsymbol{j} + 30\,\boldsymbol{k}$

   C) $-10\,\boldsymbol{i} - 10\,\boldsymbol{j} - 10\,\boldsymbol{k}$

   D) $30\,\boldsymbol{i} + 30\,\boldsymbol{j} + 30\,\boldsymbol{k}$