INTERNAL FORCES

Objective:

Use the method of sections for determining internal forces in 2-D load cases.

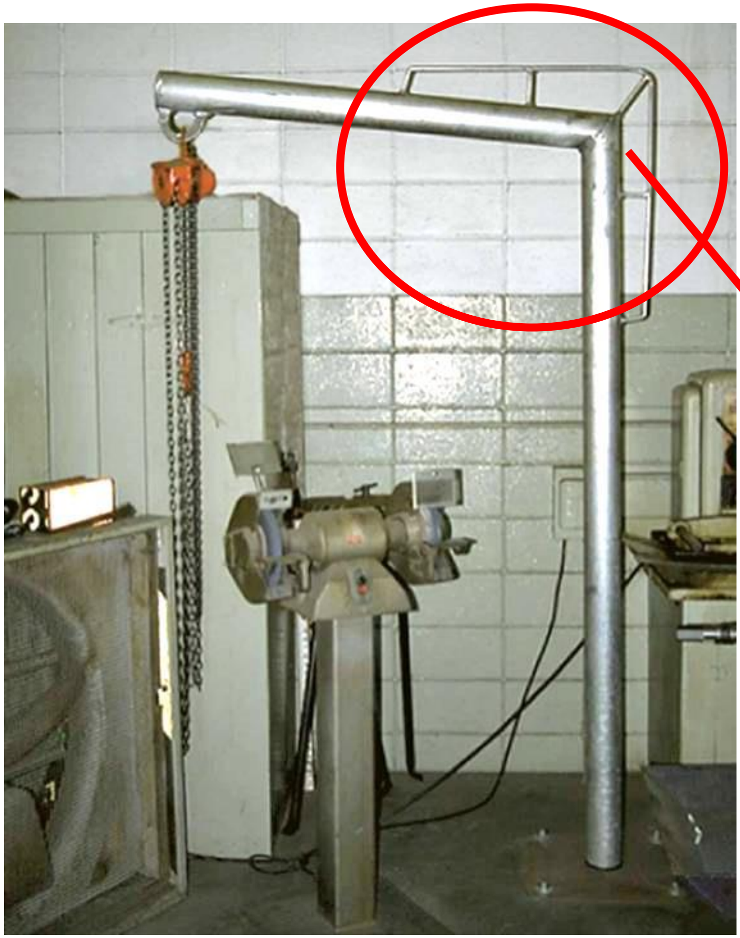
APPLICATIONS

Beams are structural members designed to support loads applied perpendicularly to their axes.

Beams can be used to support the span of bridges. They are often thicker at the supports than at the center of the span.

Why are the beams tapered?   Internal forces are important in making such a design decision. In this lesson, you will learn about these forces and how to determine them.

APPLICATIONS (continued)

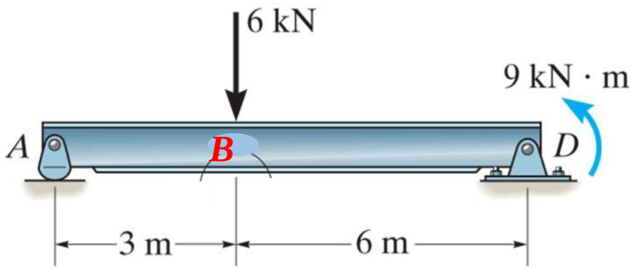


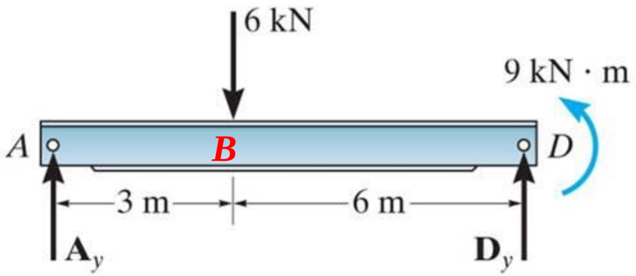
The shop crane is used to move heavy machine tools around the shop.

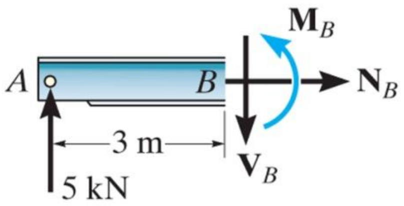
The picture shows that an additional frame around the joint is added.

Why might have this been done?

INTERNAL FORCES





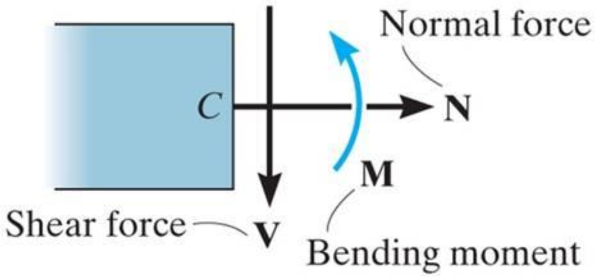


The design of any structural member requires finding the forces acting within the member to make sure the material can resist those loads.

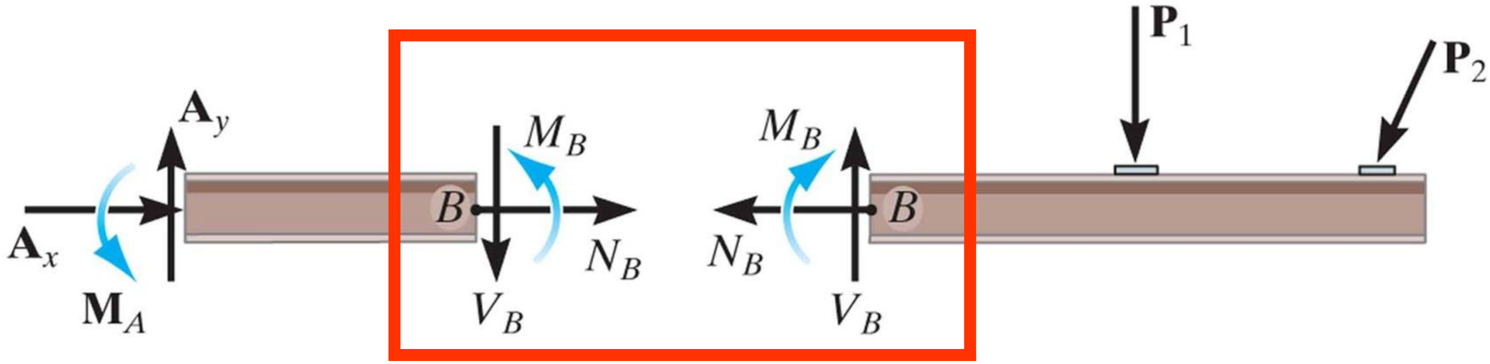
For example, we want to determine the internal forces acting on the cross section at the left of point B. But, we first need to determine the support reactions.

Then we need to cut the beam at B and draw a FBD of one of the halves of the beam. This FBD will include the internal forces acting at B. Finally, we need to solve for these unknowns using the E-of-E.

INTERNAL FORCES (continued)



In two-dimensional cases, typical internal loads are normal or axial forces (N, acting perpendicular to the section), shear forces (V, acting along the surface), and the bending moment (M). Sometimes we have internal torsional moments or torques (T).



The loads on the left and right sides of the section at B are equal in magnitude but opposite in direction. This is because when the two sides are reconnected, the net loads are zero at the section.

STEPS FOR DETERMINING INTERNAL FORCES

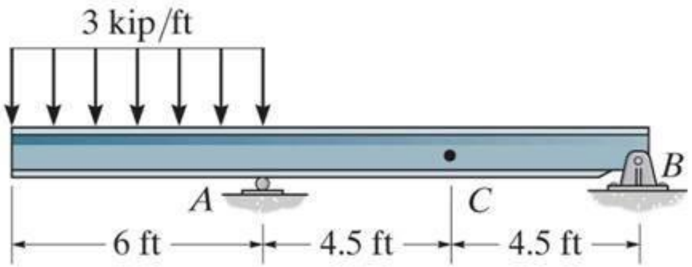
1. Take an imaginary cut at the place where you need to determine the internal loads. Then, decide which resulting section or piece will be easier to analyze.

2. If necessary, determine any support reactions or joint forces you need by drawing a FBD of the entire structure and solving for the unknown reactions.

3. Draw a FBD of the piece of the structure you've decided to analyze. Remember to show the N, V, M, and T loads at the "cut" surface.

4. Apply the E-of-E to the FBD (drawn in step 3) and solve for the unknown internal loads.

EXAMPLE I



Given: The loading on the beam.

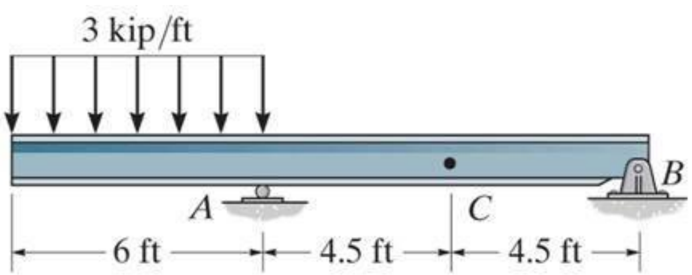
Find: The internal forces at point C.

Solution

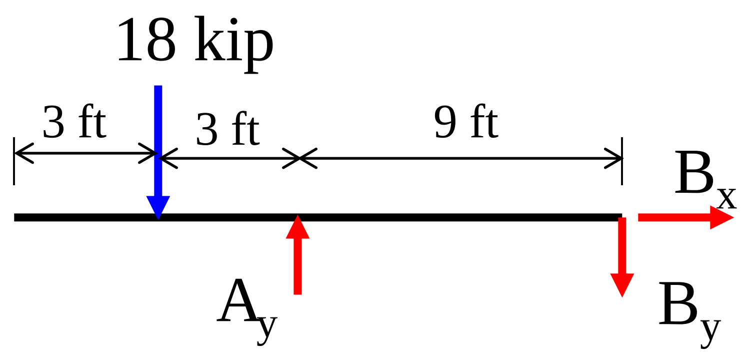
1. Plan on taking the imaginary cut at C. It will be easier to work with the right section (from the cut at C to point B) since the geometry is simpler and there is no distributed load.

EXAMPLE I (continued)

2. We need to determine B_x and B_y. Use a FBD of the entire frame and solve the E-of-E for B_y.



FBD of the entire beam:



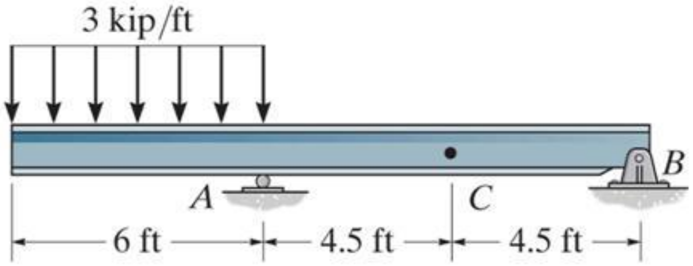
Applying the E-of-E to this FBD, we get

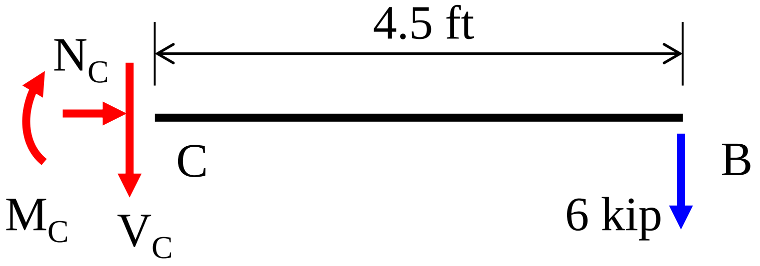
$\rightarrow + \Sigma F_x = B_x = 0;$

$\circlearrowleft + \Sigma M_A = -B_y(9) + 18(3) = 0;$ $B_y = 6$ kip

EXAMPLE I (continued)

3. Now draw a FBD of the right section. Assume directions for V_C, N_C and M_C.





4. Applying the E-of-E to this FBD, we get

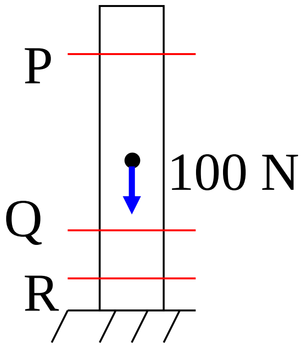
$\rightarrow + \Sigma F_x = N_C = 0;$ $\underline{N_C = 0}$

$\uparrow + \Sigma F_y = -V_C - 6 = 0;$ $\underline{V_C = -6 \text{ kip}}$

$\circlearrowleft + \Sigma M_C = -6\,(4.5) - M_C = 0\,;$ $\underline{M_C = -27 \text{ kip ft}}$

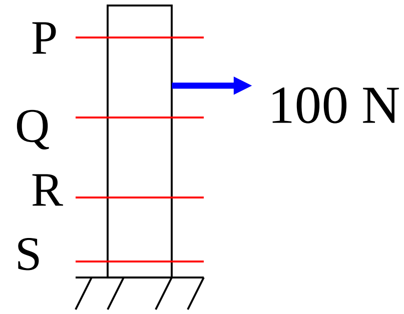
CONCEPT QUIZ

1. A column is loaded with a vertical 100 N force. At which sections are the internal loads the same?



A) P, Q, and R
B) P and Q
C) Q and R
D) None of the above.

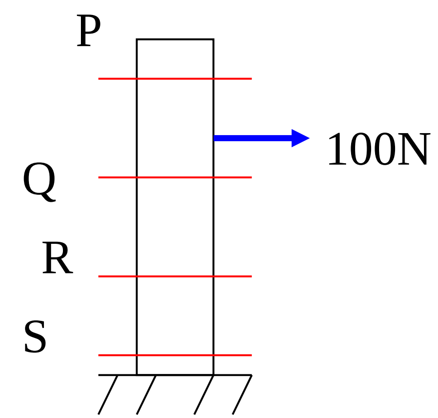
2. A beam is loaded with a horizontal 100 N force. At which section are the internal loads largest?



A) P
B) Q
C) R
D) S

CONCEPT QUIZ

3. A column is loaded with a horizontal 100 N force. At which section are the internal loads the lowest?



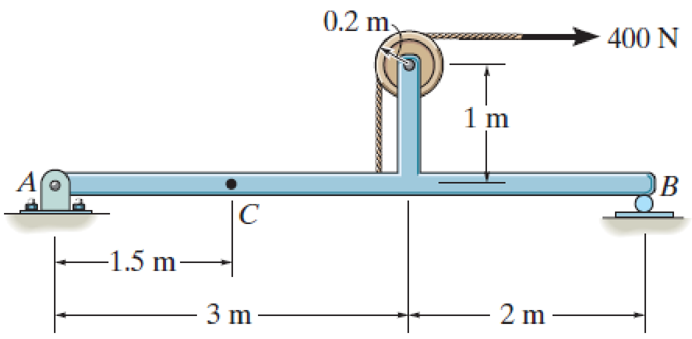
A) P

B) Q

C) R

D) S

Example II



Given: The loading on the beam.

Find: The internal forces at point C.

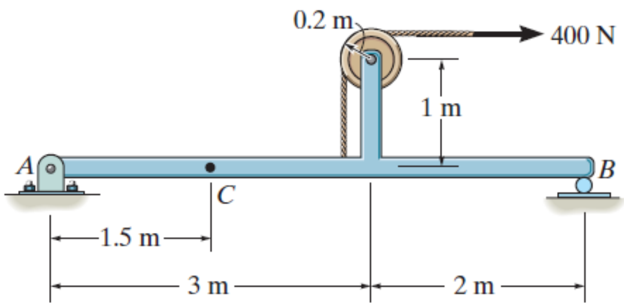
Plan: Follow the procedure!!

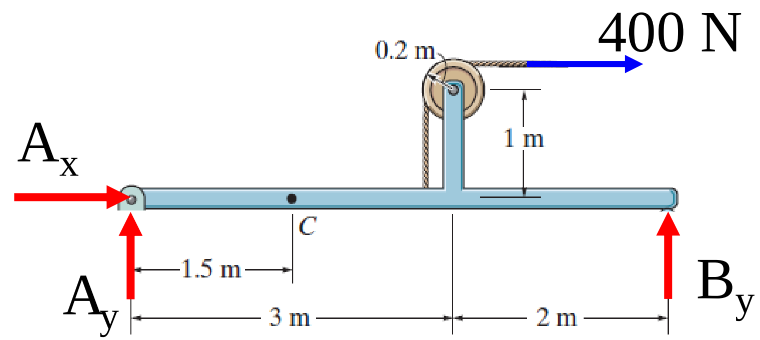
Solution:

1. Plan on taking the imaginary cut at C. It will be easier to work with the left section (point A to the cut at C) since the geometry is simpler.

Example II (continued)

2. First, we need to determine A_x and A_y using a FBD of the entire frame.





Free Body Diagram

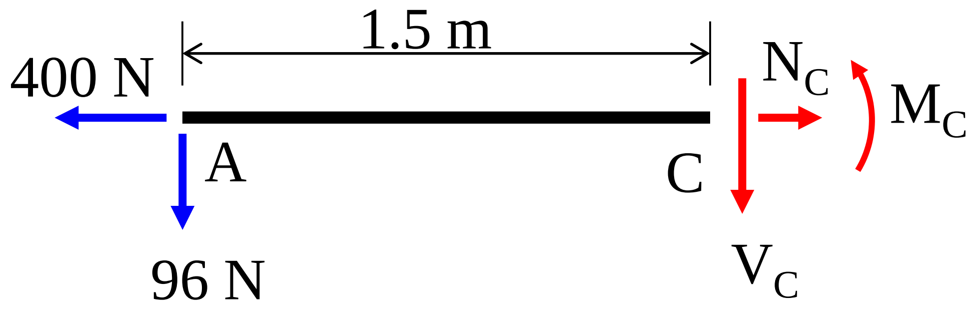
Applying the E-of-E to this FBD, we get

$\rightarrow + \Sigma F_x = A_x + 400 = 0$; $\quad A_x = -400$ N

$\circlearrowleft + \Sigma M_B = -A_y(5) - 400\ (1.2) = 0$; $\quad A_y = -96$ N

Example II (continued)

3. Now draw a FBD of the left section. Assume directions for V_C, N_C and M_C as shown.



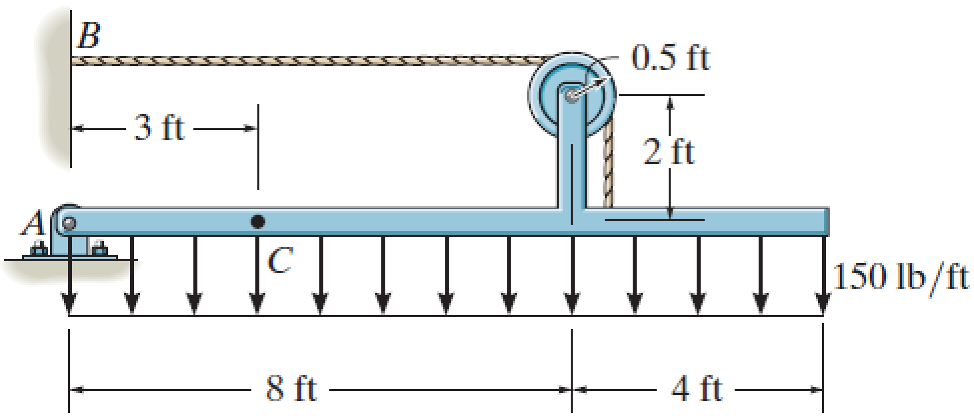
4. Applying the E-of-E to this FBD, we get

$\rightarrow + \Sigma F_x = N_C - 400 = 0;$ $\underline{N_C = 400 \text{ N}}$

$\uparrow + \Sigma F_y = -V_C - 96 = 0;$ $\underline{V_C = -96\text{N}}$

$\curvearrowleft + \Sigma M_C = 96\,(1.5) + M_C = 0\,;$ $\underline{M_C = -144 \text{ N m}}$

Example III



Given: The loading on the beam.

Find: The internal forces at point C.

Plan: Follow the procedure!!

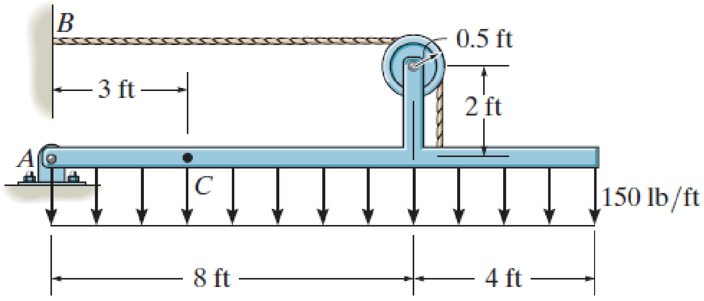
Solution:

1. Make an imaginary cut at C. Why there?
   Which section will you pick to analyze via the FBD?

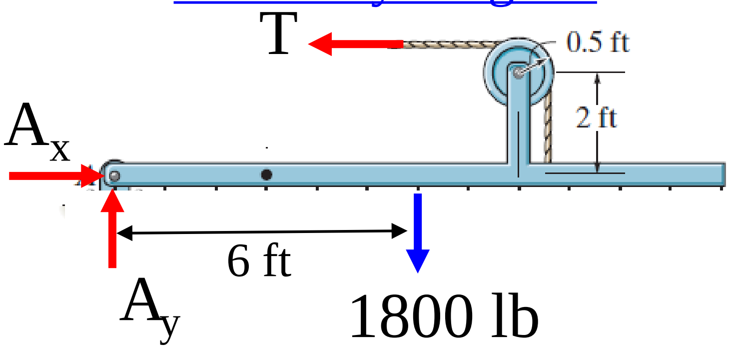
   Why will it be easier to work with segment AC?

Example III (continued)

2. Determine the reactions at A, using a FBD and the E-of-E for the entire frame.



Free Body Diagram



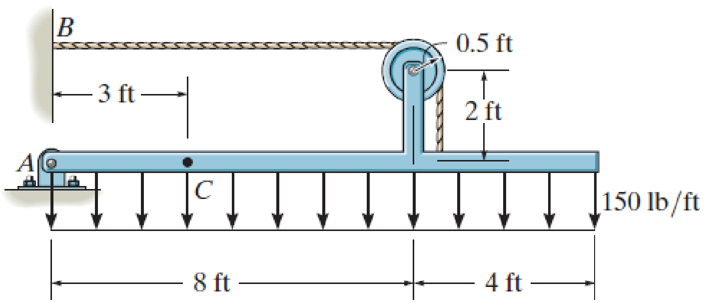
$$\circlearrowleft + \Sigma M_A = T(2.5) - 1800(6) = 0 ; \quad T = 4320 \text{ lb}$$

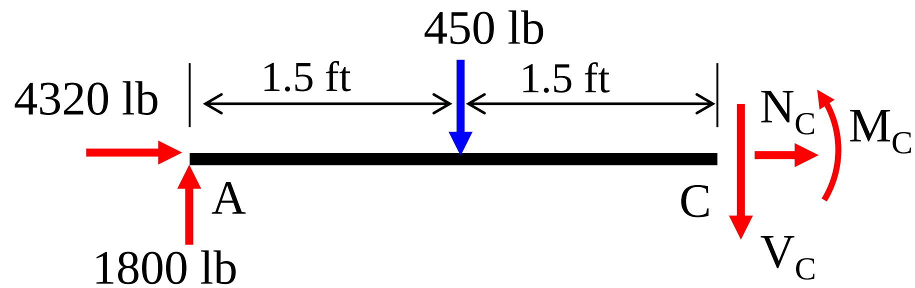
$$\rightarrow + \Sigma F_x = A_x - 4320 = 0 ; \quad A_x = 4320 \text{ lb}$$

$$\uparrow + \Sigma F_y = A_y - 1800 = 0 ; \quad A_y = 1800 \text{ lb}$$

Example III (continued)

3. A FBD of section AC is shown below.





4. Applying the E-of-E to the FBD, we get

$\rightarrow + \Sigma F_x = N_C + 4320 = 0$; $N_C = -4320$ lb

$\uparrow + \Sigma F_y = 1800 - 450 - V_C = 0$; $V_C = 1350$ lb

$\circlearrowleft + \Sigma M_C = -1800\,(3) + 450\,(1.5) + M_C = 0$; $M_C = 4725$ lb·ft