EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM & COPLANAR FORCE SYSTEMS

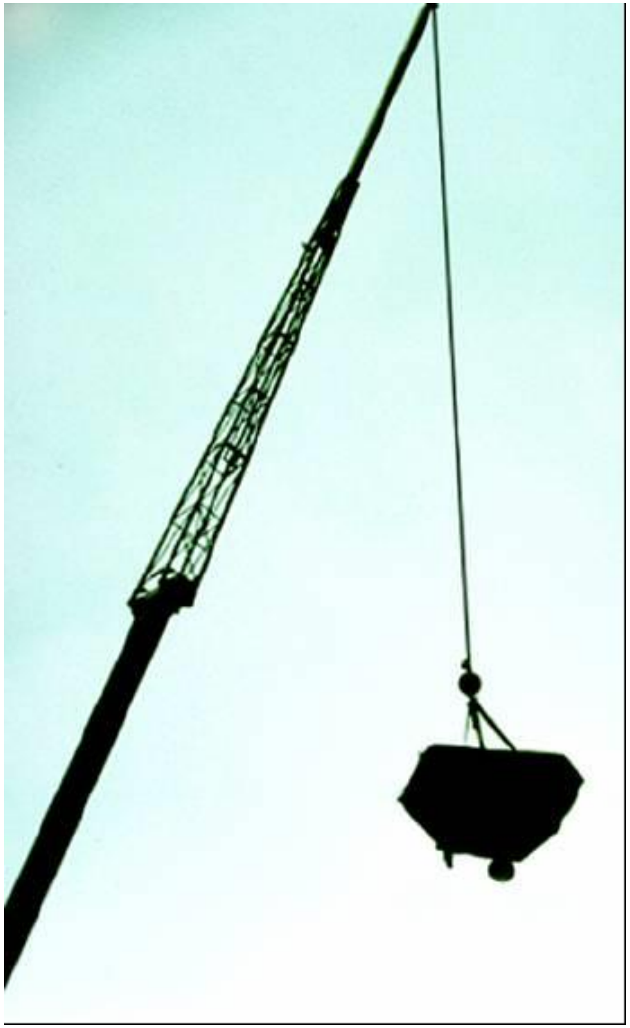
Chapter's Objectives:

Students will be able to :

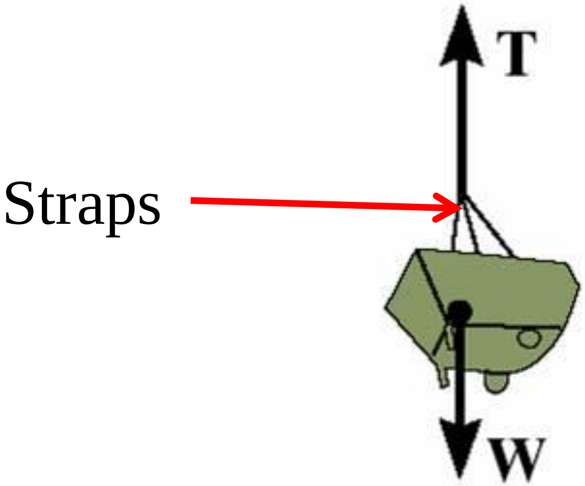
a) Draw free body diagrams (FBD), and

b) Apply equations of equilibrium to solve 2-D and 3-D problems.

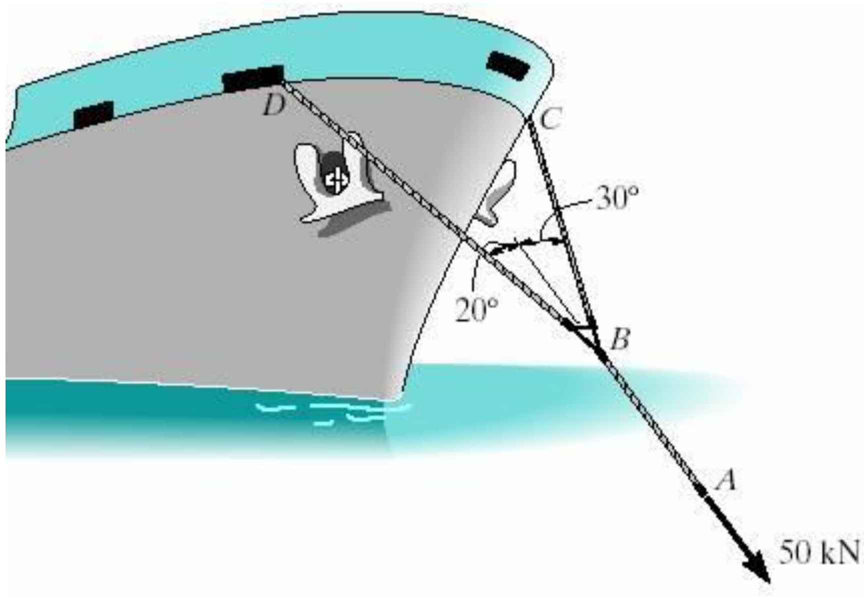
APPLICATIONS



The crane is lifting a load. To decide if the straps holding the load to the crane hook will fail, you need to know forces in the straps. How could you find those forces?

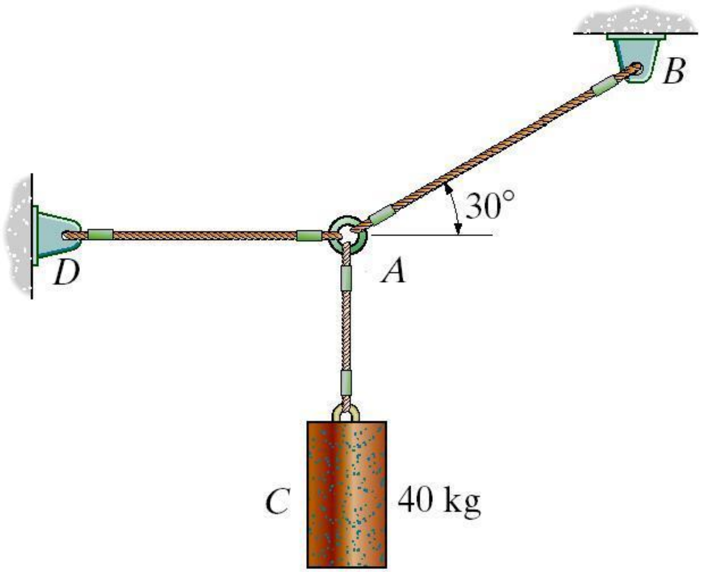


APPLICATIONS (continued)



For a given force exerted on the boat's towing pendant, what are the forces in the bridle cables? What size of cable must you use?

COPLANAR FORCE SYSTEMS (Section 3.3)



This is an example of a 2-D or coplanar force system.

If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of cylinder, you need to learn how to draw a free-body diagram and apply the equations of equilibrium.

THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

Free-body diagrams are one of the most important things for you to know how to draw and use for statics and other subjects!

What? - It is a drawing that shows all external forces acting on the particle.

Why? - It is **key** to being able to write the equations of equilibrium—which are used to solve for the unknowns (usually forces or angles).

How?

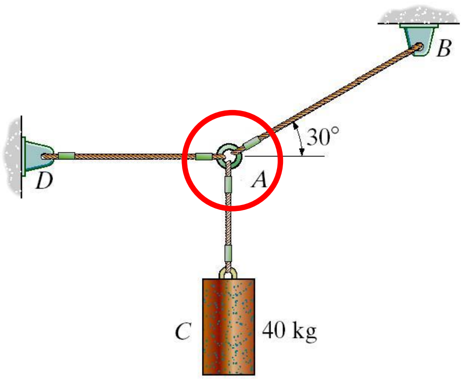
1. Imagine the particle to be isolated or cut free from its surroundings.

2. Show all the forces that act on the particle.

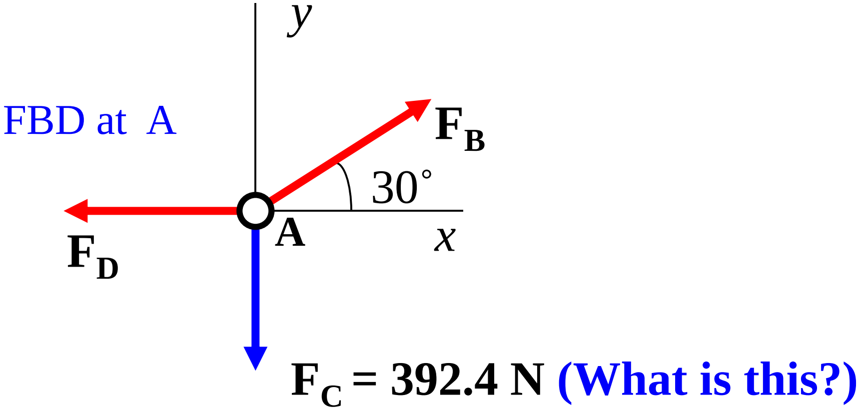
   Active forces: They want to move the particle.

   Reactive forces: They tend to resist the motion.

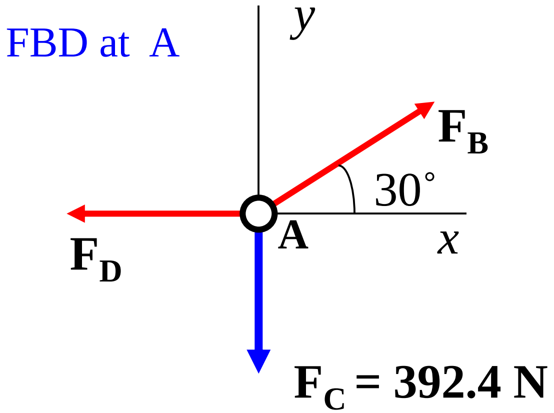
3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables.



Note : Cylinder mass = 40 Kg



EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} = 0$

or $\Sigma \boldsymbol{F} = 0$

In general, for a particle in equilibrium,

$\Sigma \boldsymbol{F} = 0$ or

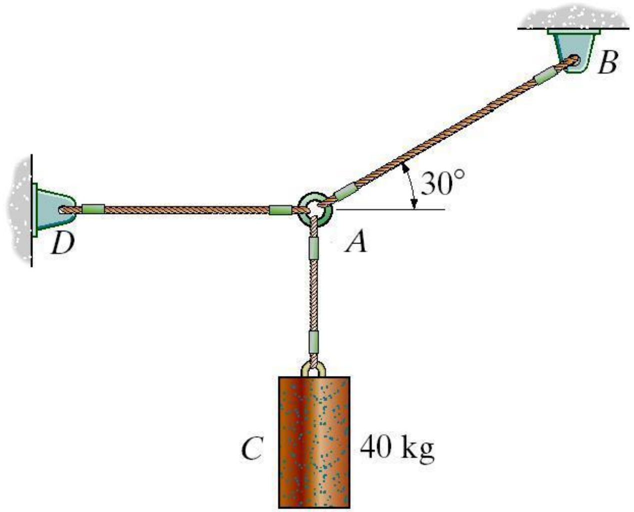
$\Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0 = 0 \boldsymbol{i} + 0 \boldsymbol{j}$ (a vector equation)

Or, written in a scalar form,

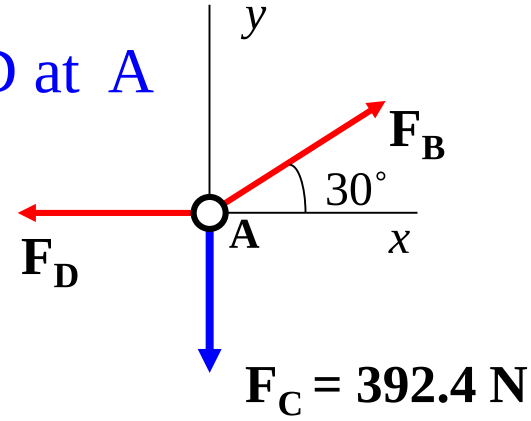
$\Sigma F_x = 0$ and $\Sigma F_y = 0$

These are two scalar equations of equilibrium (E-of-E). They can be used to solve for up to two unknowns.

EQUATIONS OF 2-D EQUILIBRIUM (continued)



FBD at A



Note : Cylinder mass = 40 Kg

Write the scalar equations of equilibrium (E-of-E):

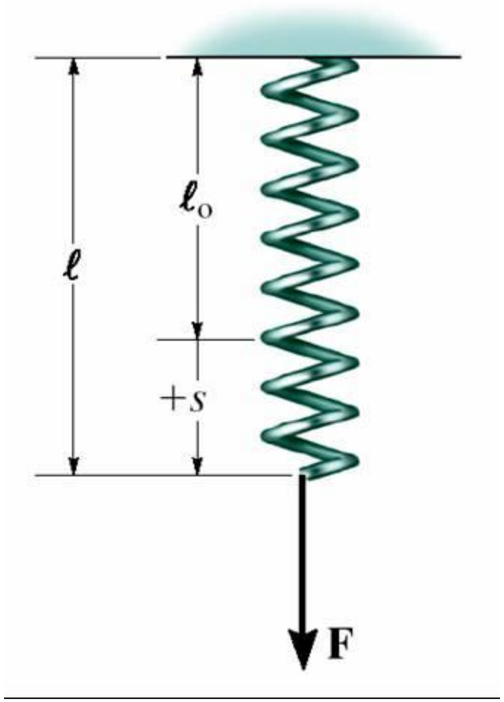
$$+\rightarrow \Sigma F_x = F_B \cos 30^\circ - F_D = 0$$

$$+\uparrow \Sigma F_y = F_B \sin 30^\circ - 392.4\text{ N} = 0$$

Solving the second equation gives: $F_B = 785$ N $\rightarrow$

From the first equation, we get: $F_D = 680$ N $\leftarrow$

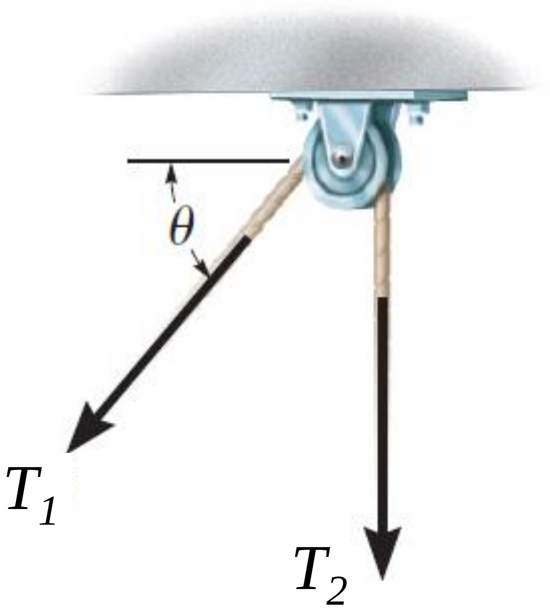
SIMPLE SPRINGS



Spring Force = spring constant × deformation of spring

or $F = k \times s$

CABLES AND PULLEYS



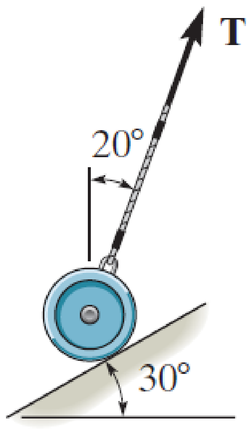
Cable is in tension

With a frictionless pulley and cable

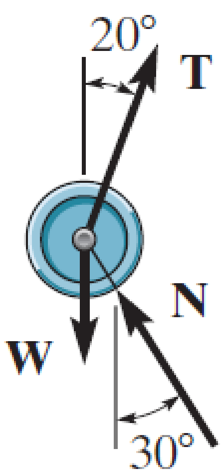
$$T_1 = T_2.$$

Cable can support *only* a tension or "pulling" force, and this force always acts in the direction of the cable.

SMOOTH CONTACT



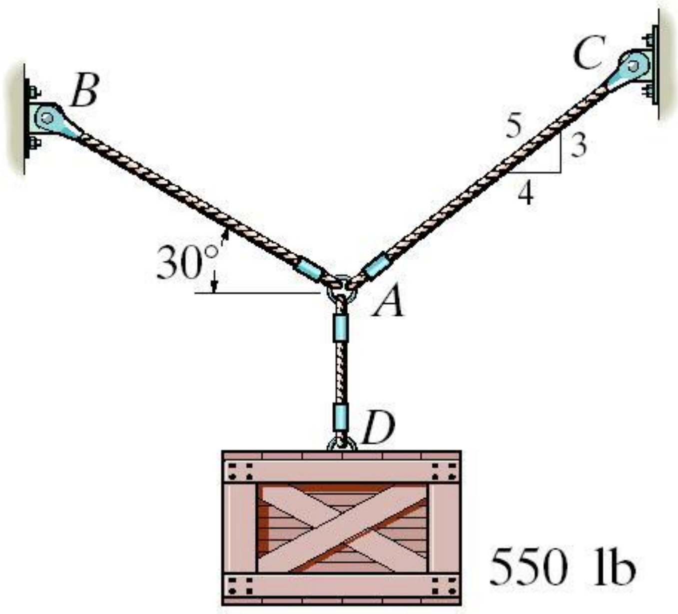
If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.



In addition to this normal force **N**, the cylinder is also subjected to its weight **W** and the force **T** of the cord.

Since these three forces are concurrent at the center of the cylinder, we can apply the equation of equilibrium to this "particle," which is the same as applying it to the cylinder.

EXAMPLE I



Given: The box weighs 550 lb and geometry is as shown.

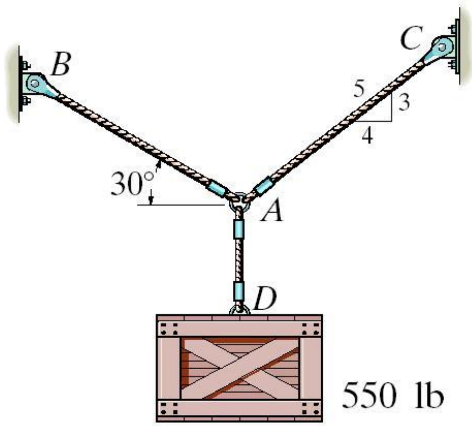
Find: The forces in the ropes AB and AC.

Plan:

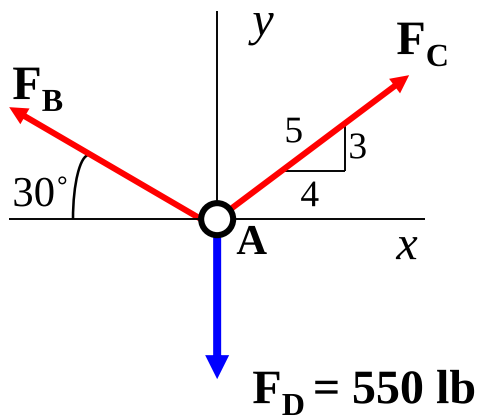
1. Draw a FBD for point A.

2. Apply the E-of-E to solve for the forces in ropes AB and AC.

EXAMPLE I (continued)



FBD at point A



Applying the scalar E-of-E at A, we get;

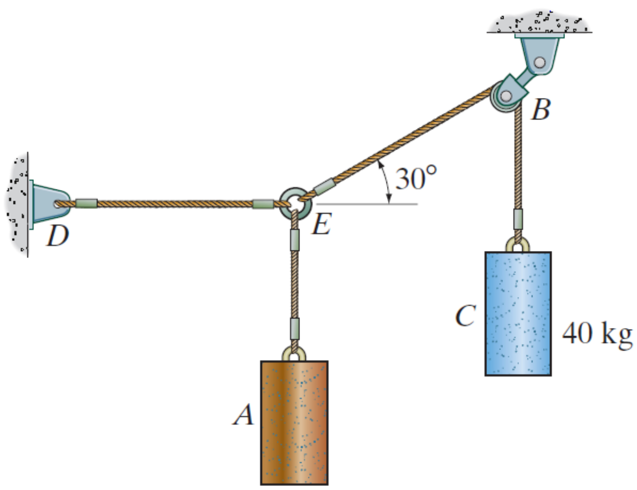
$$+\rightarrow \sum F_x = -F_B \cos 30° + F_C\,(4/5) = 0$$

$$+\uparrow \sum F_y = F_B \sin 30° + F_C\,(3/5) - 550 \text{ lb} = 0$$

Solving the above equations, we get;

$F_B = 478$ lb $\nwarrow$ and $F_C = 518$ lb $\nearrow$

EXAMPLE II



Given: The mass of cylinder C is 40 kg and geometry is as shown.

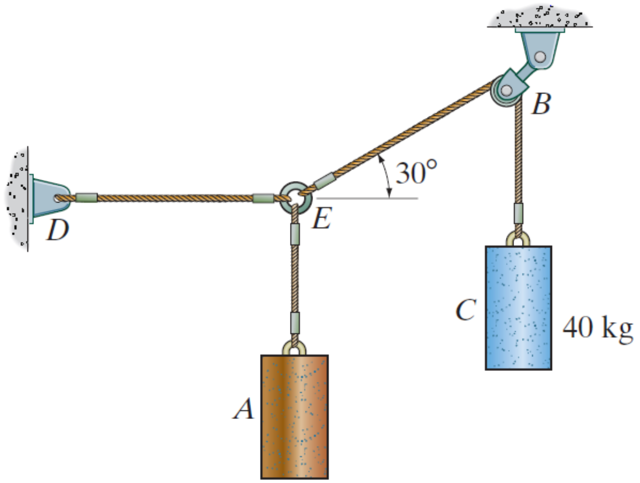
Find: The tensions in cables DE, EA, and EB.

Plan:

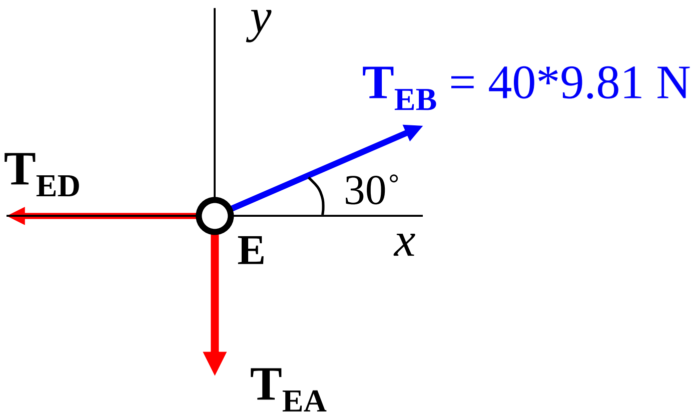
1. Draw a FBD for point E.

2. Apply the E-of-E to solve for the forces in cables DE, EA, and EB.

EXAMPLE II (continued)



FBD at point E



Applying the scalar E-of-E at E, we get;

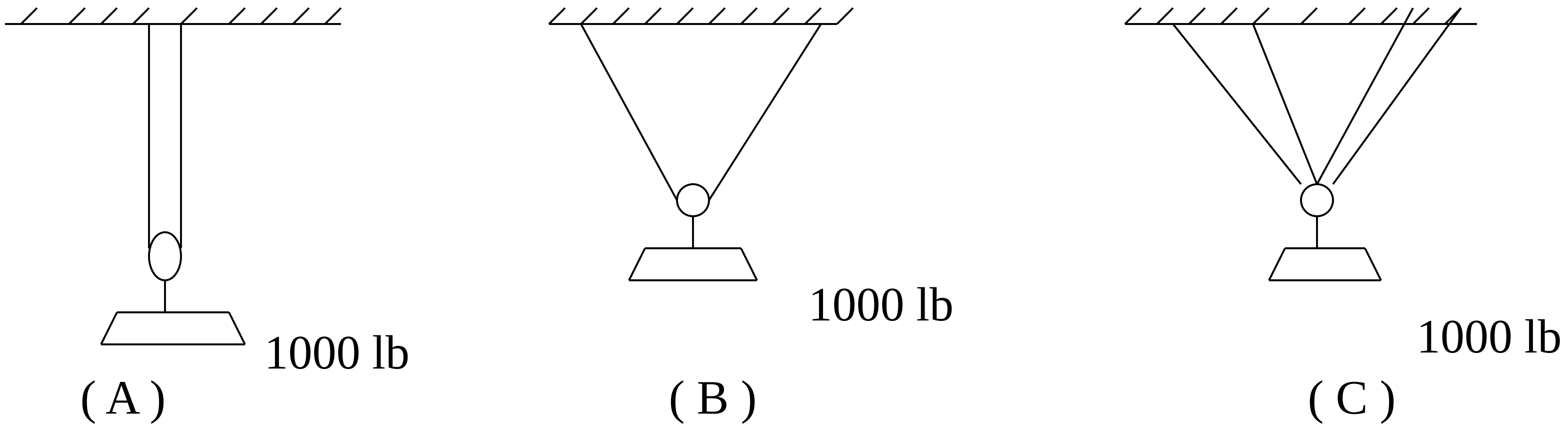
$$+\rightarrow \sum F_x = -T_{ED} + (40*9.81)\cos 30° = 0$$

$$+\uparrow \sum F_y = (40*9.81)\sin 30° - T_{EA} = 0$$

Solving the above equations, we get;

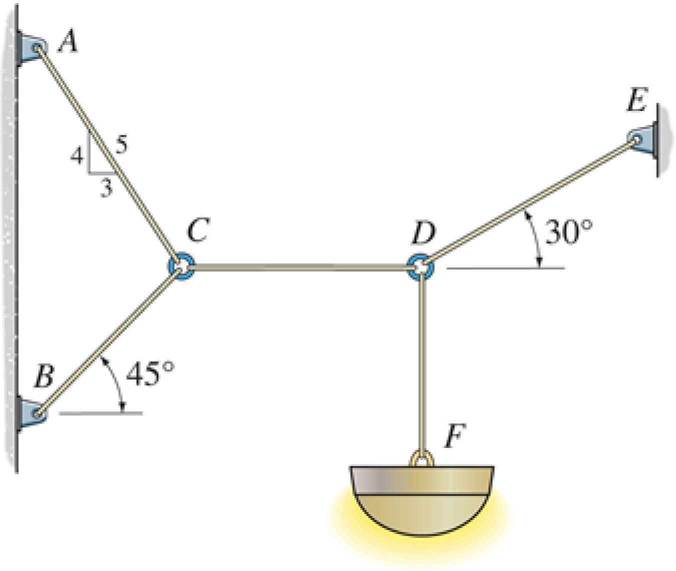
$\underline{T_{ED} = 340\text{ N}} \leftarrow$ and $\underline{T_{EA} = 196\text{ N}} \downarrow$

CONCEPT QUIZ



Assuming you know the geometry of the ropes, in which system above can you NOT determine forces in the cables? Why?

EXAMPLE III



Given: The mass of lamp is 20 kg and geometry is as shown.

Find: The force in each cable.

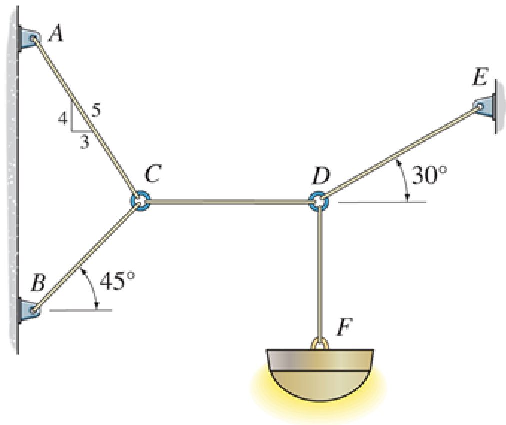
Plan:

1. Draw a FBD for Point D.

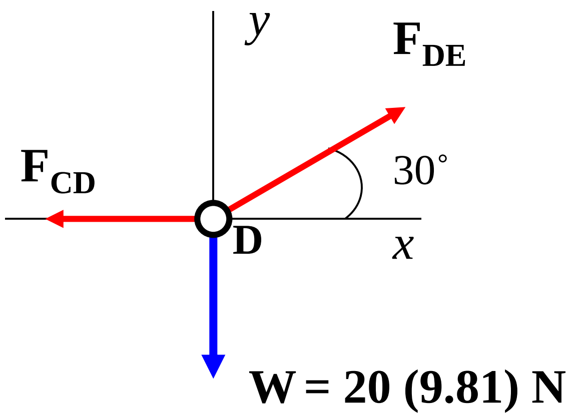
2. Apply E-of-E at Point D to solve for the unknowns (F_{CD} & F_{DE}).

3. Knowing F_{CD}, repeat this process at point C.

EXAMPLE III (continued)



FBD at point D



Applying the scalar E-of-E at D, we get;

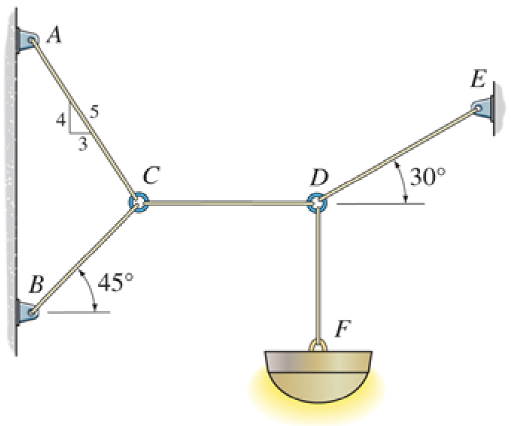
$+\uparrow \sum F_y = F_{DE} \sin 30° - 20\,(9.81) = 0$

$+\rightarrow \sum F_x = F_{DE} \cos 30° - F_{CD} = 0$

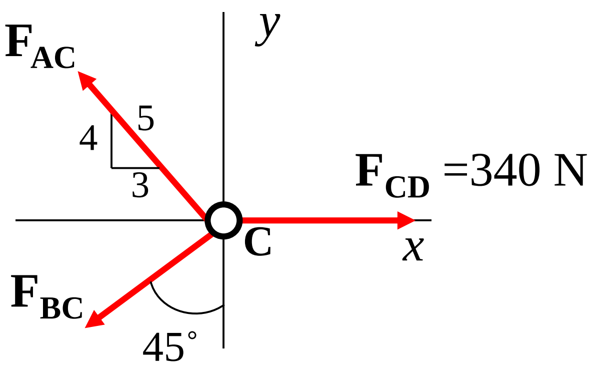
Solving the above equations, we get:

$\underline{F_{DE} = 392\text{ N}}$ $\nearrow$ and $\underline{F_{CD} = 340\text{ N}}$ $\leftarrow$

EXAMPLE III (continued)



FBD at point C



Applying the scalar E-of-E at C, we get;

$+\rightarrow \sum F_x = 340 - F_{BC} \sin 45° - F_{AC}\,(3/5) = 0$

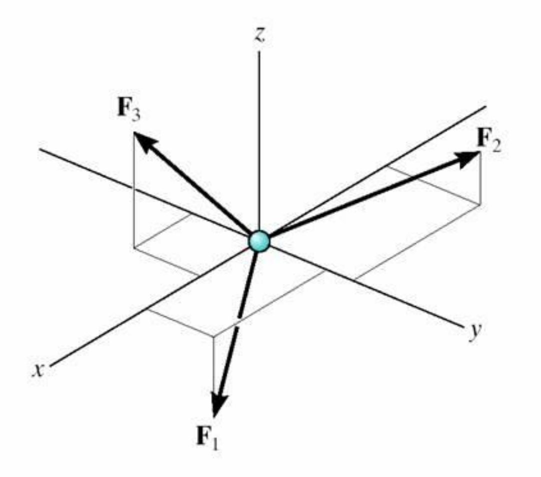
$+\uparrow \sum F_y = F_{AC}\,(4/5) - F_{BC} \cos 45° = 0$

Solving the above equations, we get;

$F_{BC} = 275$ N $\swarrow$ and $F_{AC} = 243$ N $\nwarrow$

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \mathbf{F} = 0$).



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\,\mathbf{i} + (\Sigma F_y)\,\mathbf{j} + (\Sigma F_z)\,\mathbf{k} = 0$$

This vector equation will be satisfied only when

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

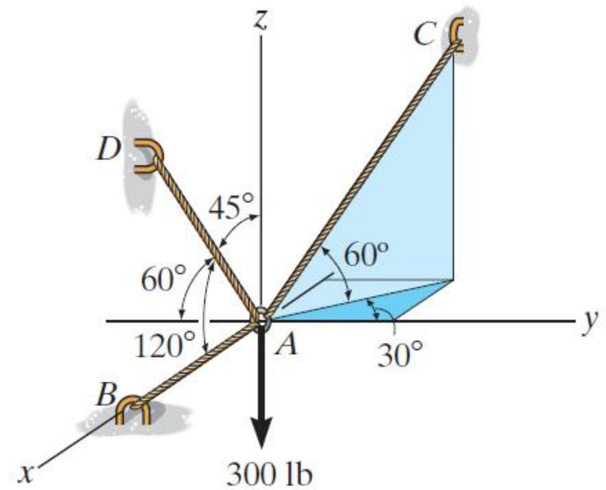
These equations are the three scalar equations of equilibrium. They are valid for any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE IV

Given: The four forces and geometry shown.

Find: The tension developed in cables AB, AC, and AD.

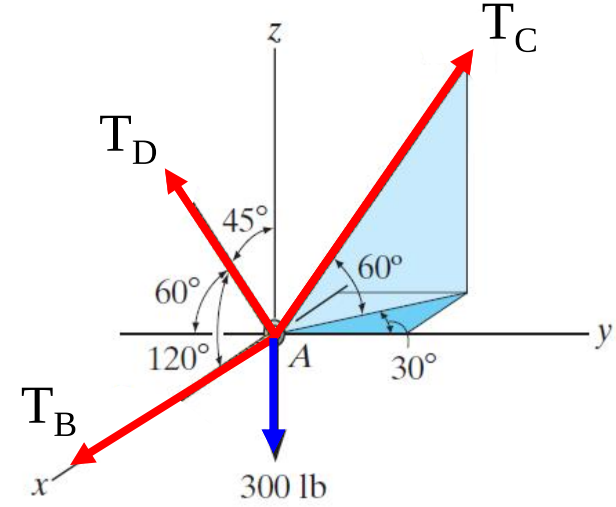
Plan:



1) Draw a FBD of particle A.
2) Write the unknown cable forces T_B, T_C , and T_D in Cartesian vector form.
3) Apply the three equilibrium equations to solve for the tension in cables.

EXAMPLE IV (continued)

Solution:



$\boldsymbol{T_B} = T_B\ \boldsymbol{i}$

$\boldsymbol{T_C} = -(T_C \cos 60°) \sin 30°\ \boldsymbol{i}$
$\quad + (T_C \cos 60°) \cos 30°\ \boldsymbol{j}$
$\quad + T_C \sin 60°\ \boldsymbol{k}$

$\boldsymbol{T_C} = T_C\,(-0.25\ \boldsymbol{i}\ + 0.433\ \boldsymbol{j} + 0.866\ \boldsymbol{k}\ )$

$\boldsymbol{T_D} = T_D \cos 120°\ \boldsymbol{i} + T_D \cos 120°\ \boldsymbol{j} + T_D \cos 45°\ \boldsymbol{k}$

$\boldsymbol{T_D} = T_D\,(-0.5\ \boldsymbol{i} - 0.5\ \boldsymbol{j} + 0.7071\ \boldsymbol{k}\ )$

$\boldsymbol{W} = -300\ \boldsymbol{k}$

EXAMPLE IV (continued)

Applying equilibrium equations:

$$\Sigma \boldsymbol{F_R} = 0 = T_B\, \boldsymbol{i} + T_C\,(-\,0.25\, \boldsymbol{i} + 0.433\, \boldsymbol{j} + 0.866\, \boldsymbol{k}) + T_D\,(-\,0.5\, \boldsymbol{i} - 0.5\, \boldsymbol{j} + 0.7071\, \boldsymbol{k}) - 300\, \boldsymbol{k}$$

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$$\Sigma F_x = T_B - 0.25\, T_C - 0.5\, T_D = 0 \qquad (1)$$

$$\Sigma F_y = 0.433\, T_C - 0.5\, T_D = 0 \qquad (2)$$

$$\Sigma F_z = 0.866\, T_C + 0.7071\, T_D - 300 = 0 \qquad (3)$$

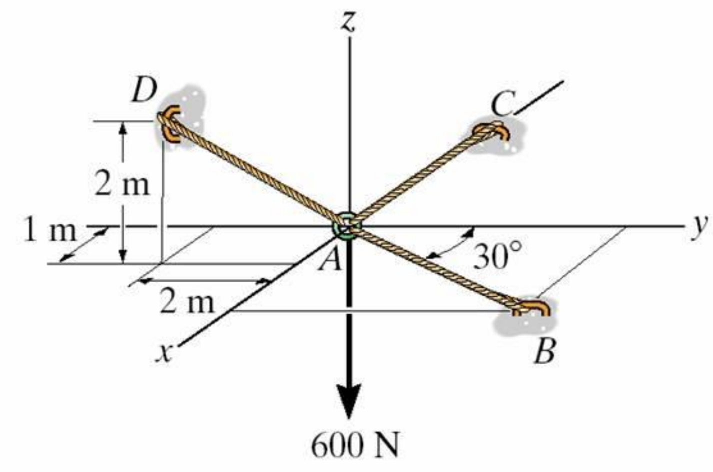
Using (2) and (3), we can determine $T_C = 203$ lb, $T_D = 176$ lb

Substituting T_C and T_D into (1), we can find $T_B = 139$ lb

EXAMPLE V

Given: A 600 N load is supported by three cords with the geometry as shown.

Find: The tension in cords AB, AC and AD.



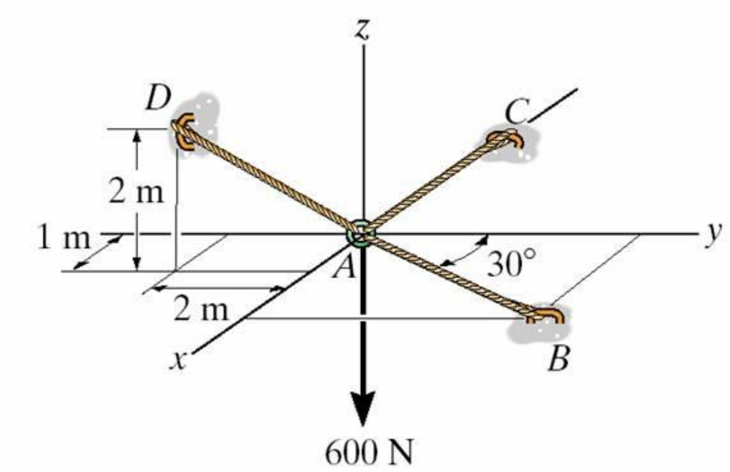
Plan:

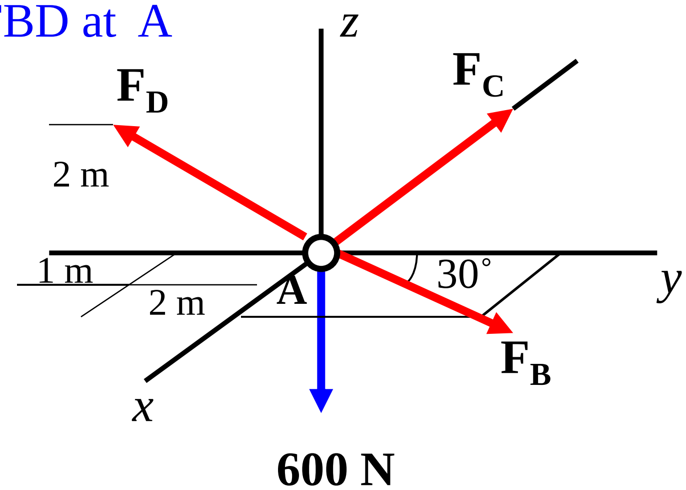
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

2) Represent each force in its Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

EXAMPLE V (continued)





$$\boldsymbol{F_B} = F_B\,(\sin 30°\ \boldsymbol{i} + \cos 30°\ \boldsymbol{j})\ N$$

$$= \{0.5\ F_B\ \boldsymbol{i} + 0.866\ F_B\ \boldsymbol{j}\}\ N$$

$$\boldsymbol{F_C} = -\,F_C\ \boldsymbol{i}\ \ N$$

$$\boldsymbol{F_D} = F_D\,(\boldsymbol{r_{AD}}/r_{AD})$$

$$= F_D\,\{\,(1\ \boldsymbol{i} - 2\ \boldsymbol{j} + 2\ \boldsymbol{k}) / (1^2 + 2^2 + 2^2)^{½}\ \}\ N$$

$$= \{\ 0.333\ F_D\ \boldsymbol{i} - 0.667\ F_D\ \boldsymbol{j} + 0.667\ F_D\ \boldsymbol{k}\ \}\ N$$

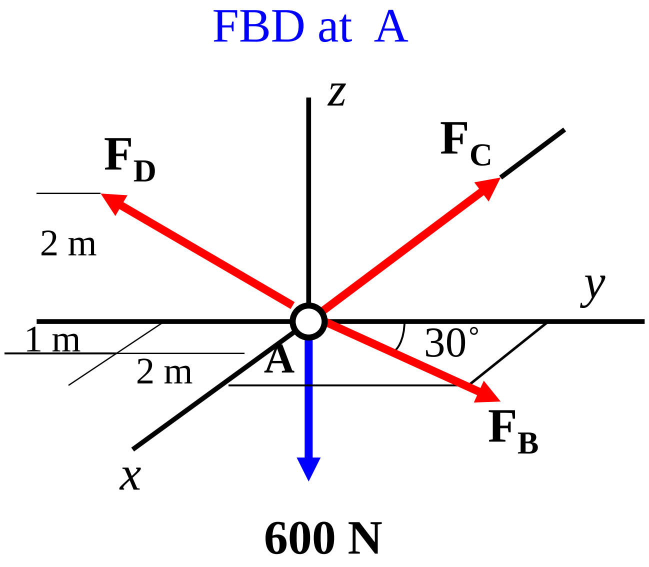
EXAMPLE V (continued)

Now equate the respective ***i***, ***j***, and ***k*** components to zero.

$\sum F_x = 0.5\ F_B - F_C + 0.333\ F_D = 0$

$\sum F_y = 0.866\ F_B - 0.667\ F_D = 0$

$\sum F_z = 0.667\ F_D - 600 = 0$



Solving the three simultaneous equations yields

$F_C = 646$ N (since it is positive, it is as assumed, e.g., in tension)

$F_D = 900$ N

$F_B = 693$ N

CONCEPT QUIZ

1. In 3-D, when you know the direction of a force but not its magnitude, how many unknowns corresponding to that force remain?

   A) One B) Two C) Three D) Four

2. What if you know the magnitude, but not the direction?

   A) One B) Two C) Three D) Four

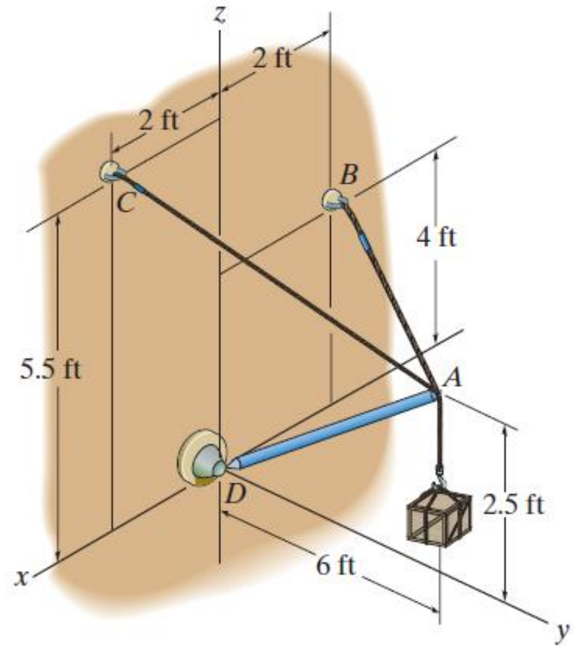
3. In 3-D, when you don't know the direction and the magnitude of a force, how many unknowns do you have corresponding to that force?

   A) One B) Two C) Three D) Four

EXAMPLE VI

Given: A 400 lb crate, as shown, is in equilibrium and supported by two cables and a strut AD.

Find: Magnitude of the tension in each of the cables and the force developed along strut AD.



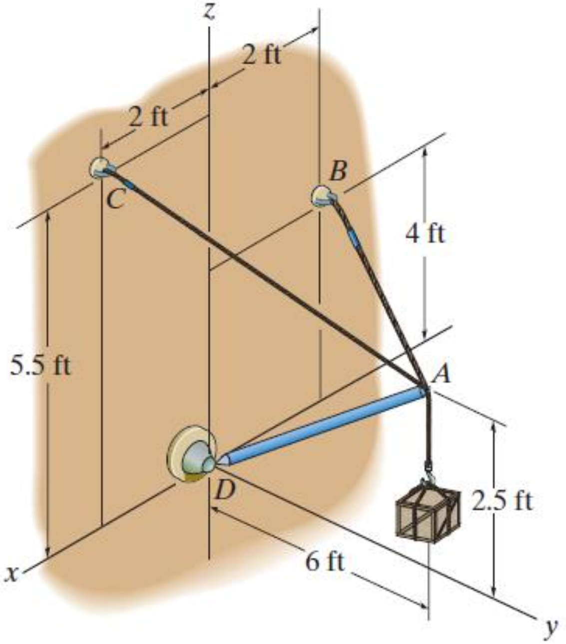
Plan:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

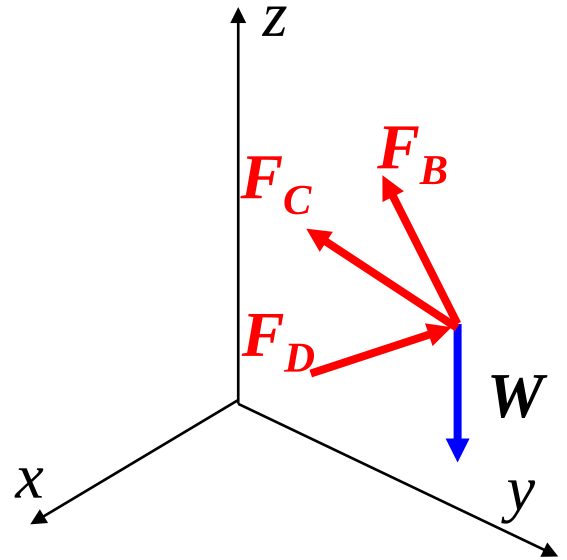
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

EXAMPLE VI (continued)



FBD of Point A



$\boldsymbol{W}$ = weight of crate = - 400 $\boldsymbol{k}$ lb

$\boldsymbol{F_B}$ = $F_B(\boldsymbol{r_{AB}}/r_{AB})$ = F_B {(– 4 $\boldsymbol{i}$ – 12 $\boldsymbol{j}$ + 3 $\boldsymbol{k}$) / (13)} lb

$\boldsymbol{F_C}$ = $F_C(\boldsymbol{r_{AC}}/r_{AC})$ = F_C {(2 $\boldsymbol{i}$ – 6 $\boldsymbol{j}$ + 3 $\boldsymbol{k}$) / (7)}lb

$\boldsymbol{F_D}$ = $F_D(\boldsymbol{r_{AD}}/r_{AD})$ = F_D {(12 $\boldsymbol{j}$ + 5 $\boldsymbol{k}$) / (13)}lb

EXAMPLE VI (continued)

The particle A is in equilibrium, hence

$\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$$\sum F_x = -(4/13)\,F_B + (2/7)\,F_C = 0 \qquad (1)$$

$$\sum F_y = -(12/13)\,F_B - (6/7)\,F_C + (12/13)\,F_D = 0 \qquad (2)$$

$$\sum F_z = (3/13)\,F_B + (3/7)\,F_C + (5/13)\,F_D - 400 = 0 \qquad (3)$$

Solving the three simultaneous equations gives the forces

$F_B = 274$ lb

$F_C = 295$ lb

$F_D = 547$ lb