3-D EQUILIBRIUM EQUATIONS

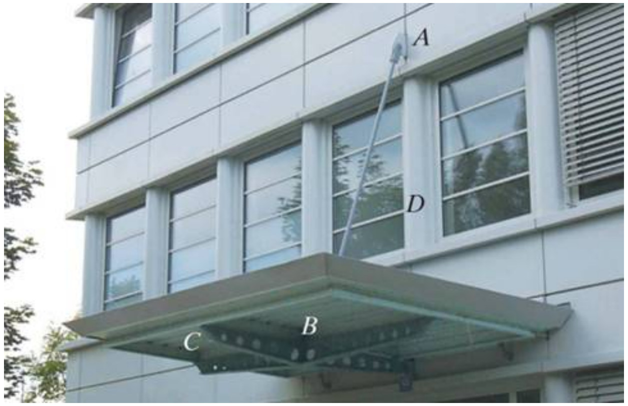
Objectives:

Students will be able to:

a) Identify support reactions in 3-D and draw a free-body diagram, and,

b) Apply the equations of equilibrium.

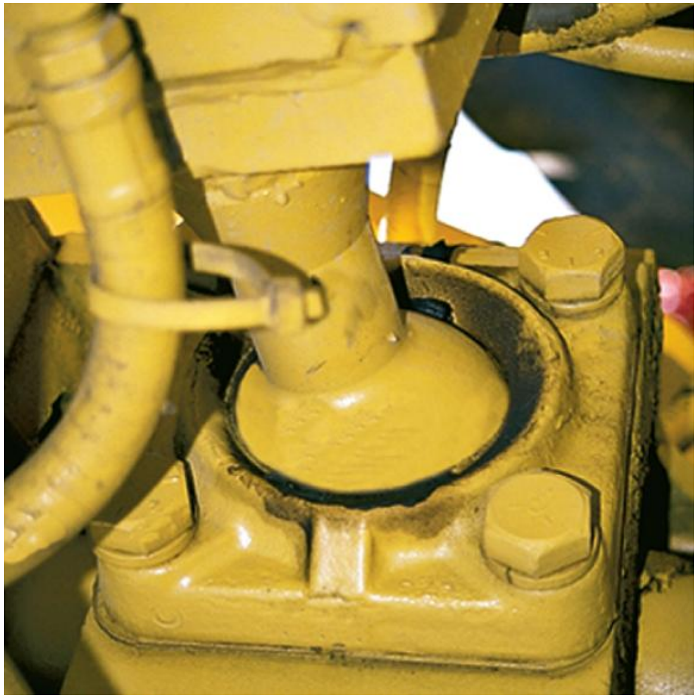
APPLICATIONS

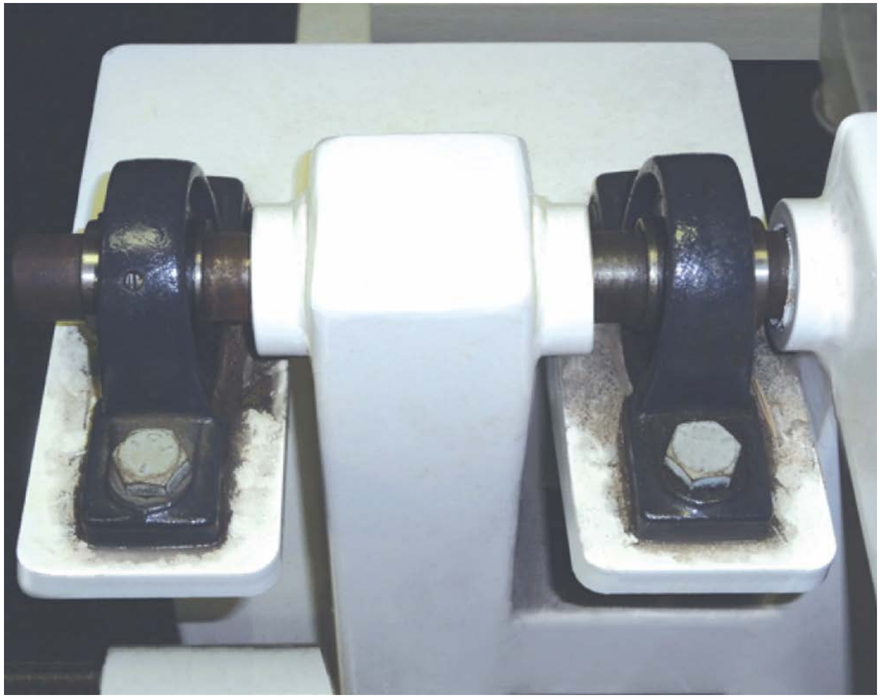


The tie rod from point A is used to support the overhang at the entrance of a building. It is pin connected to the wall at A and to the center of the overhang B.

If A is moved to a lower position D, will the force in the rod change or remain the same? By making such a change without understanding if there is a change in forces, failure might occur.

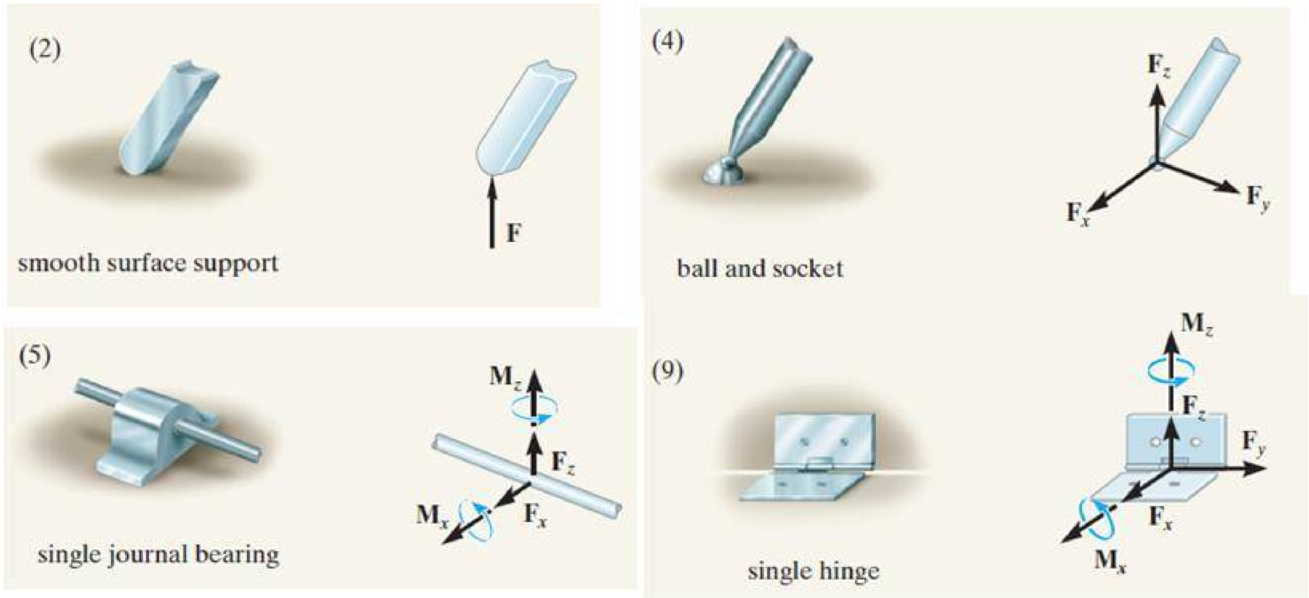
APPLICATIONS





Ball-and-socket joints and journal bearings are often used in mechanical systems. To design the joints or bearings, the support reactions at these joints and the loads must be determined.

SUPPORT REACTIONS IN 3-D



A few examples of supports are shown above. Other support reactions are given in your textbook (Table 5-2).

As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

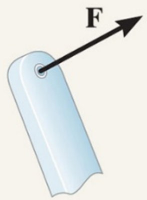
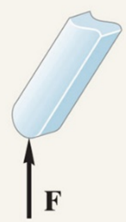
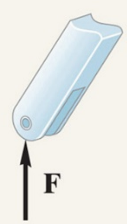
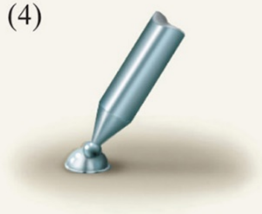
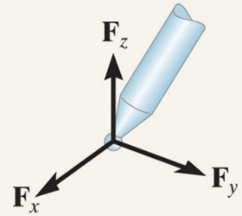
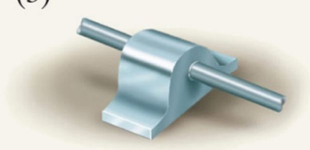
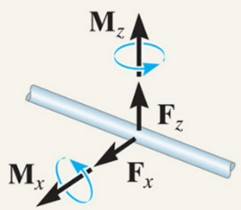
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

continued

TABLE 5–2 Continued

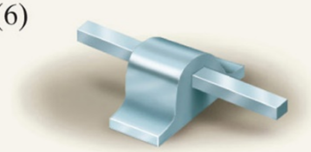
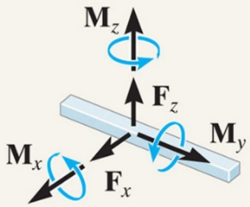
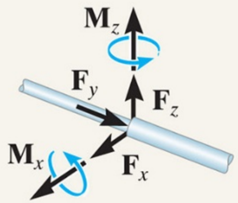
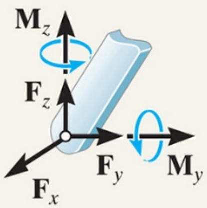
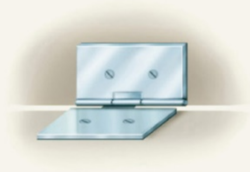
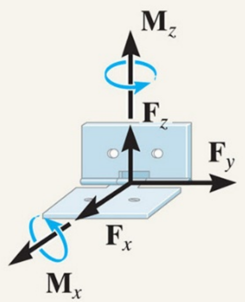
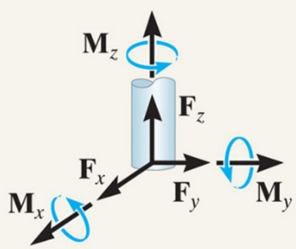
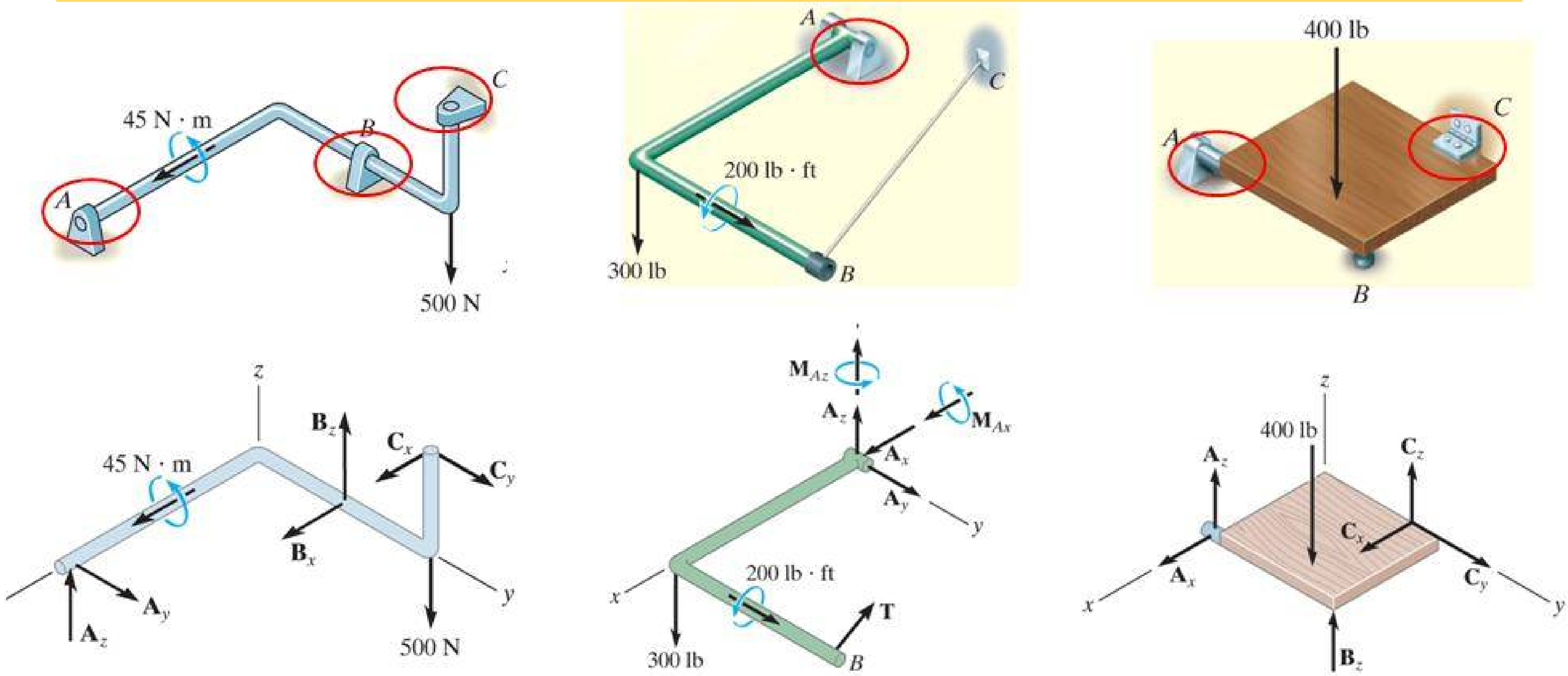
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

IMPORTANT NOTE



It is usually preferred to use two or more properly aligned bearings or hinges. In these cases, only force reactions are generated and no moment reactions are created.

EQUATIONS OF EQUILIBRIUM

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$.

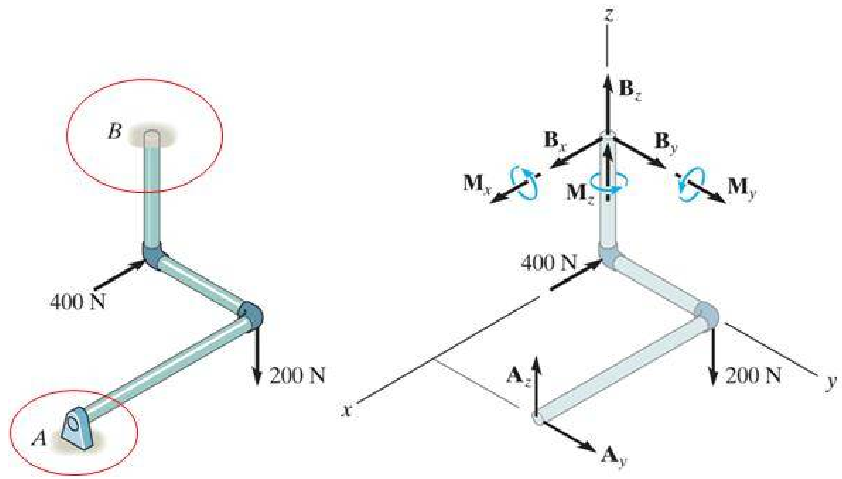
These two vector equations can be written as six scalar equations of equilibrium (E-of-E). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Any forces passing through the point where moments are taken do not appear in the moment equation.

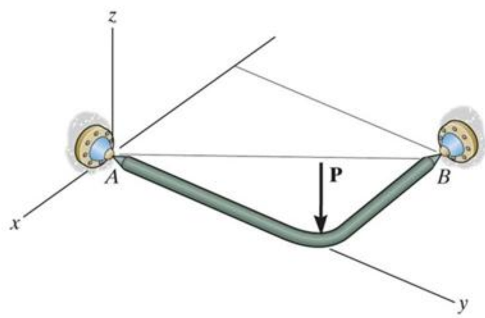
CONSTRAINTS AND STATICAL DETERMINACY

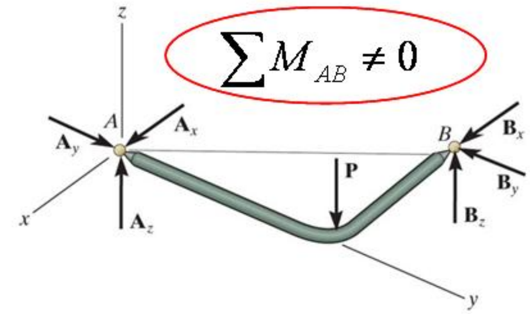


When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

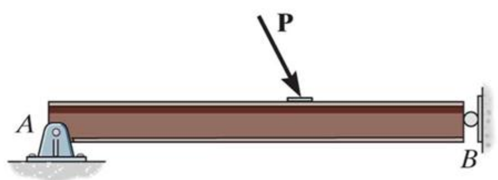
Are statically indeterminate structures used in practice? Why or why not?

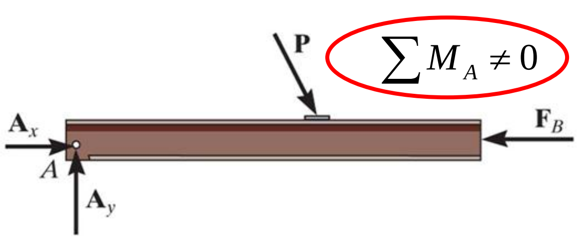
IMPROPER CONSTRAINTS





Here, while we have 6 unknowns, there is nothing restricting rotation about the AB axis!

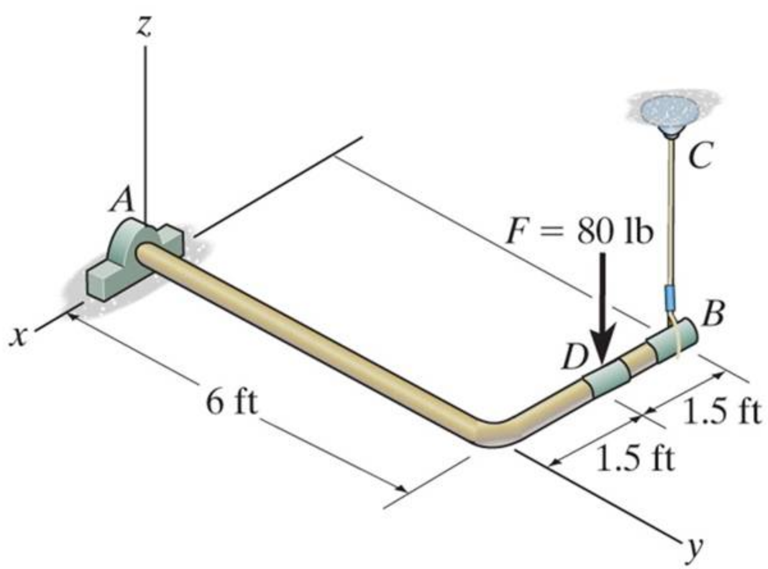




In some cases, there may be as many unknown reactions as there are equations of equilibrium.

However, if the supports are not properly constrained, the body may become unstable for some loading cases.

EXAMPLE I



Given: The rod, supported by thrust bearing at A and cable BC, is subjected to an 80 lb force.

Find: Reactions at the thrust bearing A and cable BC.

Plan:

a) Use the established x, y and z-axes.

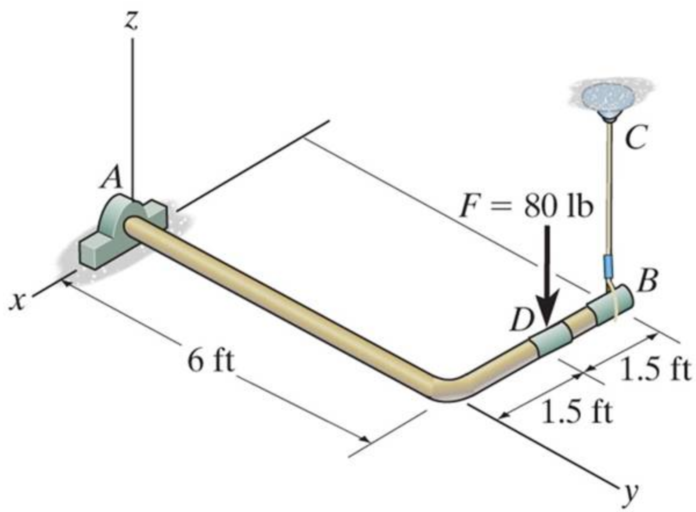
b) Draw a FBD of the rod.

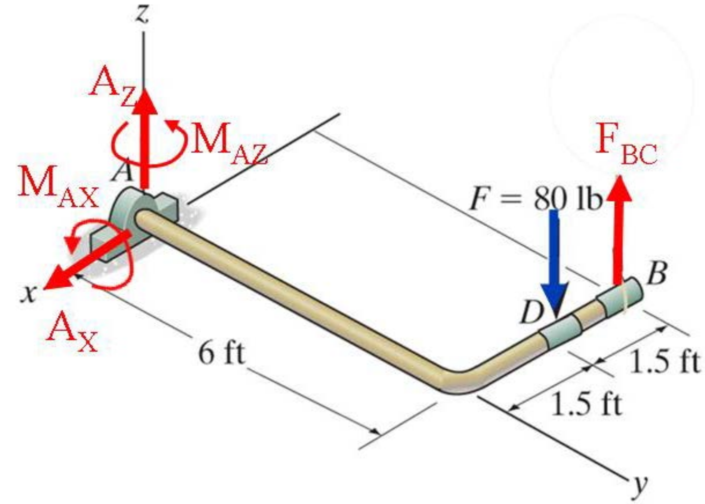
c) Write the forces using scalar equations.

d) Apply scalar equations of equilibrium to solve for the unknown forces.

EXAMPLE I (continued)

FBD of the rod:





Applying scalar equations of equilibrium in appropriate order, we get

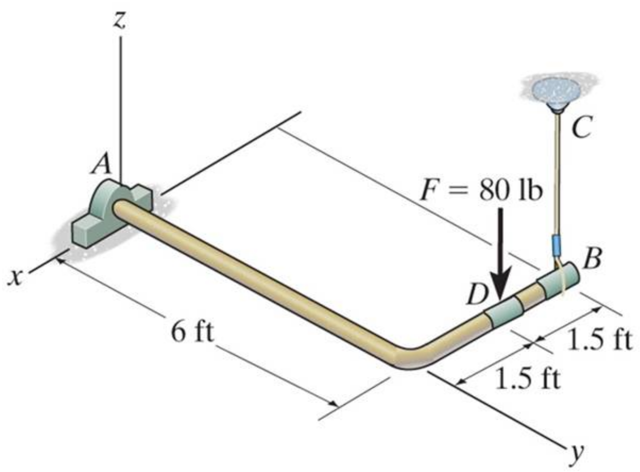
$\sum F_X = A_X = 0;$ $\underline{A_X = 0}$

$\sum F_Z = A_Z + F_{BC} - 80 = 0;$

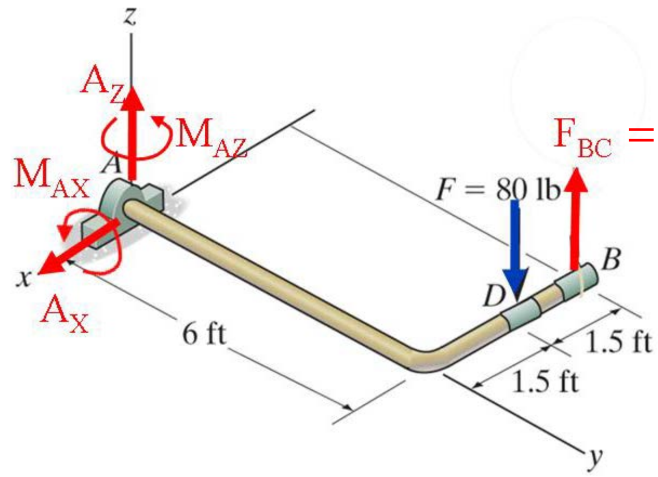
$\sum M_Y = -80(1.5) + F_{BC}(3.0) = 0;$

Solving the last two equations: $\underline{F_{BC} = 40 \text{ lb}}$, $\underline{A_Z = 40 \text{ lb}}$

EXAMPLE I (continued)



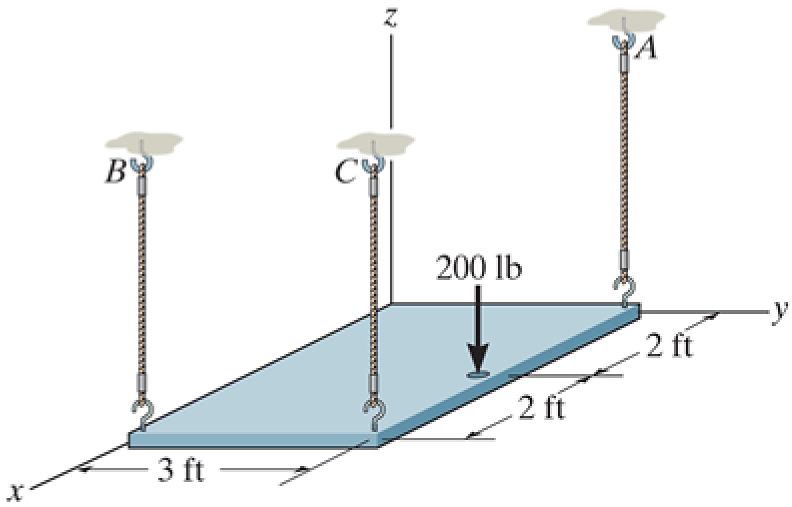
FBD of the rod



$M_X = (M_A)_X + 40\,(6) - 80\,(6) = 0$; $(M_A)_X = 240$ lb ft CCW

$\sum M_Z = (M_A)_Z = 0$; $(M_A)_Z = 0$

EXAMPLE II



Given: The uniform plate has a weight of 500 lb, supported by three cables.

Find: The tension in each of the supporting cables.

Plan:

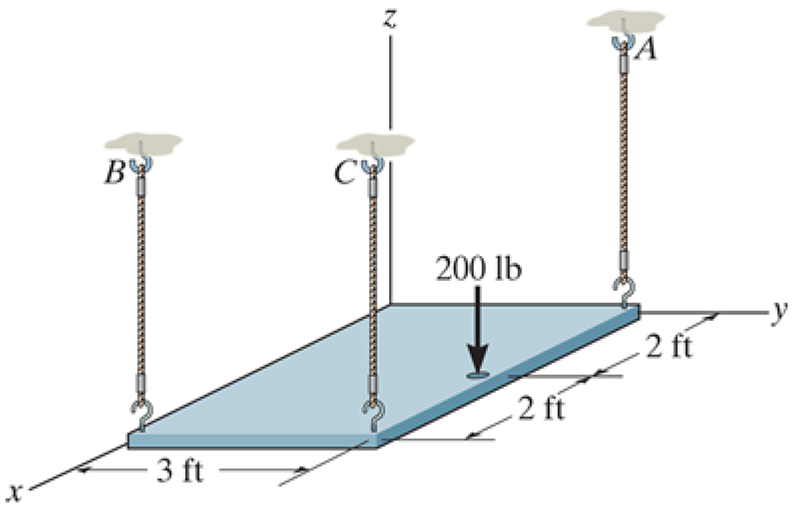
a) Use established x, y and z-axes.

b) Draw a FBD of the plate.

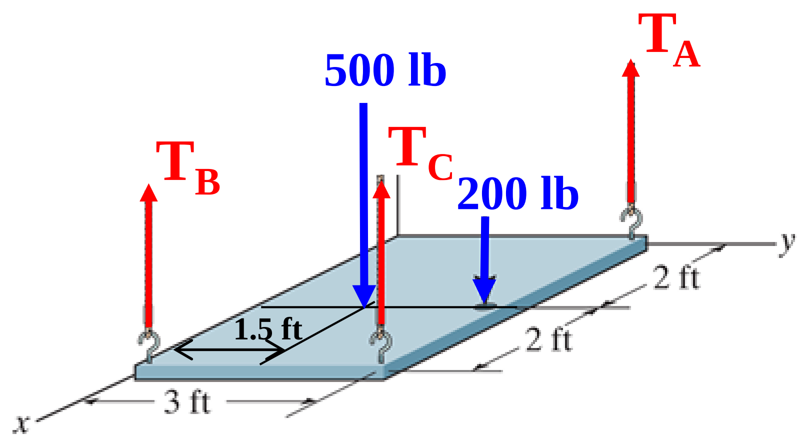
c) Write the forces using scalar equations.

d) Apply scalar equations of equilibrium to solve for the unknown forces.

EXAMPLE II (continued)



FBD of the plate:



Applying scalar equations of equilibrium:

$$\Sigma F_z = T_A + T_B + T_C - 200 - 500 = 0 \qquad (1)$$

$$\Sigma M_x = T_A (3) + T_C (3) - 500 (1.5) - 200 (3) = 0 \qquad (2)$$

$$\Sigma M_y = - T_B (4) - T_C (4) + 500 (2) + 200 (2) = 0 \qquad (3)$$

EXAMPLE II (continued)

Using Eqs. (2) and (3), express T_A and T_B in terms of T_C:

Eq. (2) $\Rightarrow$ $T_A = 450 - T_C$

Eq. (3) $\Rightarrow$ $T_B = 350 - T_C$

Substituting the results into Eq. (1) & solving for T_C

Eq. (1) $\Rightarrow$ $(450 - T_C) + (350 - T_C) + T_C - 200 - 500 = 0$

$\underline{T_C = 100 \text{ lb}}\uparrow$

$\underline{T_A = 350 \text{ lb}}\uparrow$ and $\underline{T_B = 250 \text{ lb}}\uparrow$

CONCEPT QUIZ

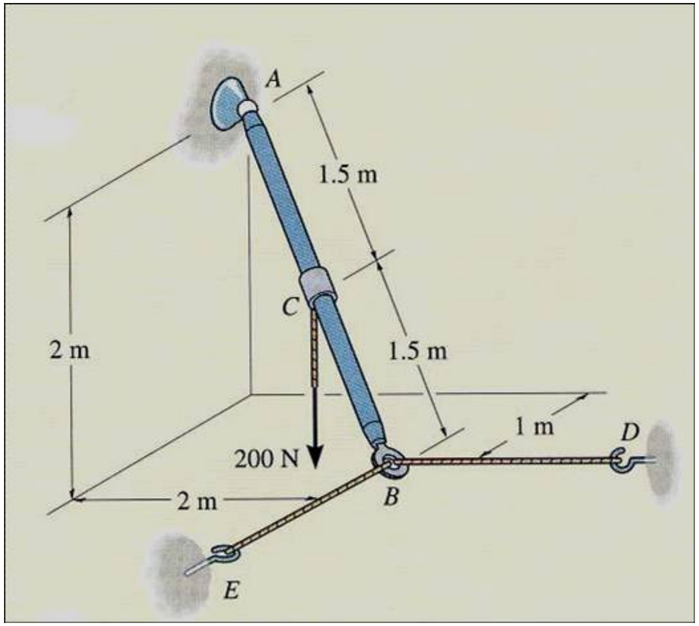
1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?

   A) Five force and one moment reaction

   B) Five force reactions

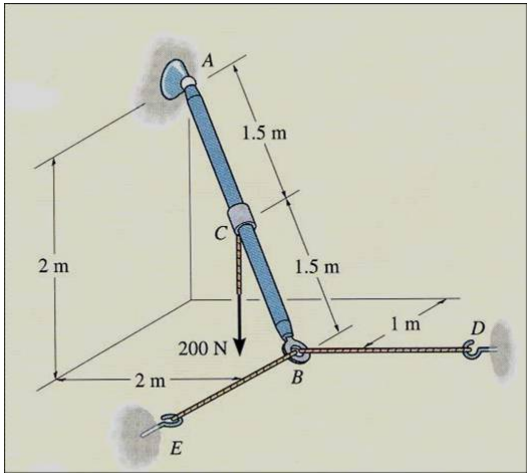
   C) Three force and three moment reactions

   D) Four force and two moment reactions



CONCEPT QUIZ (continued)

2. If an additional couple moment in the vertical direction is applied to rod AB at point C, then what will happen to the rod?



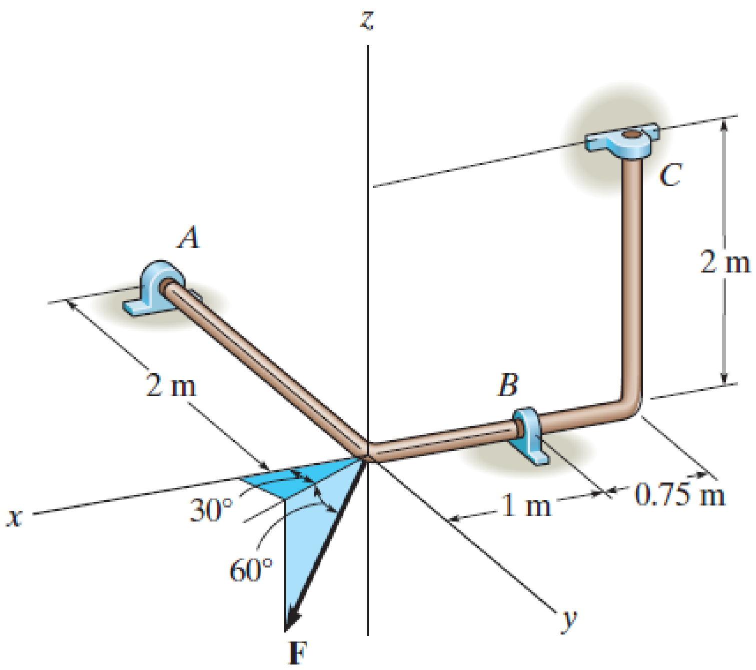
A) The rod remains in equilibrium as the cables provide the necessary support reactions.

B) The rod remains in equilibrium as the ball-and-socket joint will provide the necessary resistive reactions.

C) The rod becomes unstable as the cables cannot support compressive forces.

D) The rod becomes unstable since a moment about AB cannot be restricted.

EXAMPLE III



Given: A bent rod is supported by smooth journal bearings at A, B, and C. F=800 N. Assume the rod is properly aligned.

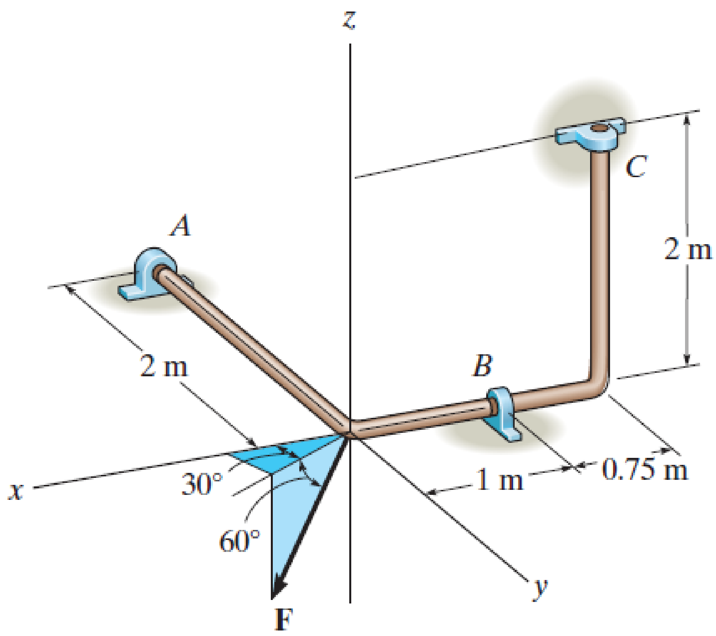
Find: The reactions at all the supports.

Plan:

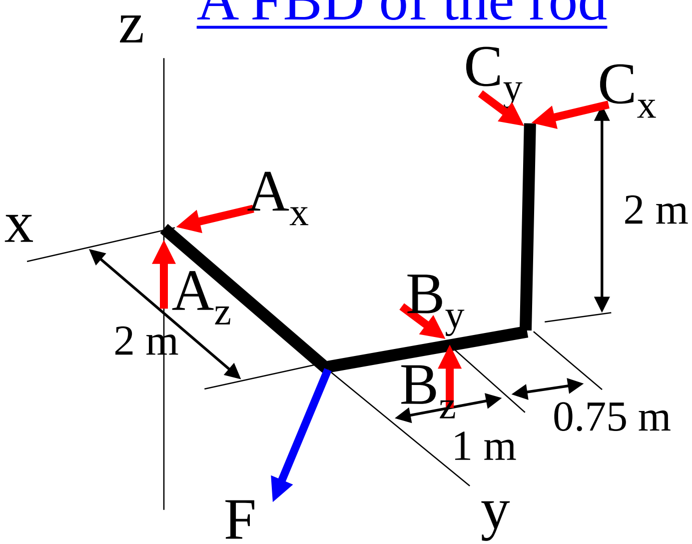
a) Draw a FBD of the rod.

b) Apply scalar equations of equilibrium to solve for the unknowns.

EXAMPLE III (continued)



A FBD of the rod



The x, y and z components of force F are

$F_x = (800 \cos 60°) \cos 30° = 346.4$ N

$F_y = (800 \cos 60°) \sin 30° = 200$ N

$F_z = 800 \sin 60° = 692.8$ N

$\boldsymbol{F} = 346.4\, \boldsymbol{i} + 200\, \boldsymbol{j} - 692.8\, \boldsymbol{k}$

EXAMPLE III (continued)

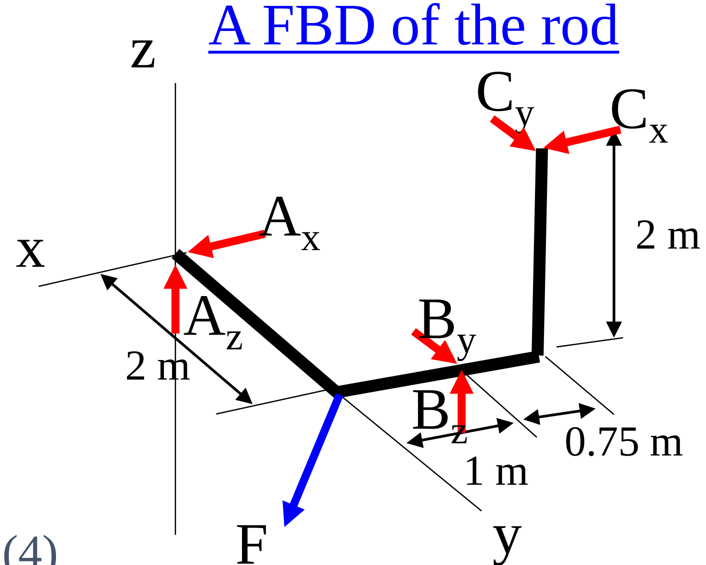
Applying scalar equations of equilibrium, we get

$$\Sigma F_x = A_x + C_x + 346.4 = 0 \qquad (1)$$

$$\Sigma F_y = 200 + B_y + C_y = 0 \qquad (2)$$

$$\Sigma F_z = A_z + B_z - 692.8 = 0 \qquad (3)$$

A FBD of the rod



$$\Sigma M_x = -C_y(2) + B_z(2) - 692.8(2) = 0 \qquad (4)$$

$$\Sigma M_y = B_z(1) + C_x(2) = 0 \qquad (5)$$

$$\Sigma M_z = -C_y(1.75) - C_x(2) - B_y(1) - 346.4(2) = 0 \qquad (6)$$

Recall

$\boldsymbol{F} = 346.4\,\boldsymbol{i} + 200\,\boldsymbol{j} - 692.8\,\boldsymbol{k}$

Solving Eqs. (1) to (6),

$A_x = 400$ N, $B_y = 600$ N, $C_x = 53.6$ N

$A_z = 800$ N, $B_z = -107$ N, $C_y = 800$ N

QUIZ

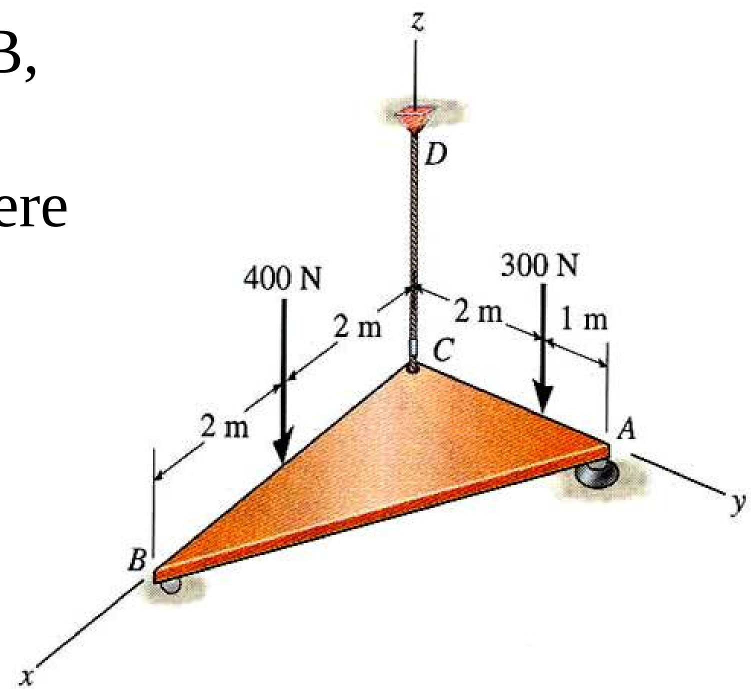
1. A plate is supported by a ball-and-socket joint at A, a roller joint at B, and a cable at C. How many unknown support reactions are there in this problem?

   A) Four forces and two moments

   B) Six forces

   C) Five forces

   D) Four forces and one moment



QUIZ

2. What will be the easiest way to determine the force reaction B_Z ?

   A) Scalar equation $\sum F_Z = 0$

   B) Vector equation $\sum \boldsymbol{M_A} = 0$

   C) Scalar equation $\sum M_Z = 0$

   D) Scalar equation $\sum M_Y = 0$