TRUSSES

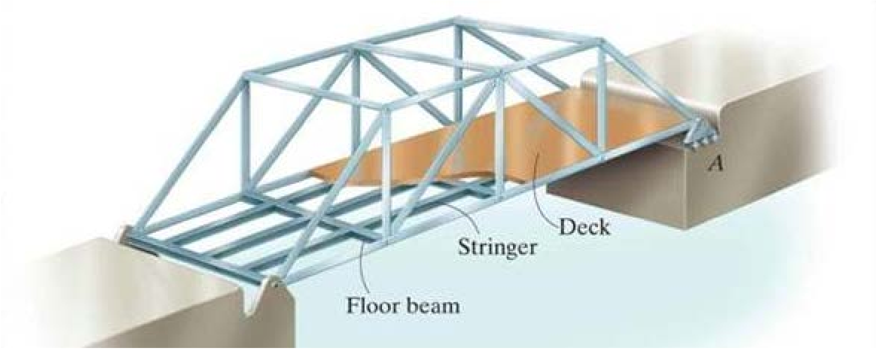
<u>Objectives</u>:

a) Define a simple truss.

b) Determine forces in members of a simple truss.

c) Identify zero-force members.

d) Forces in truss members using the method of joints and method of sections.



APPLICATIONS

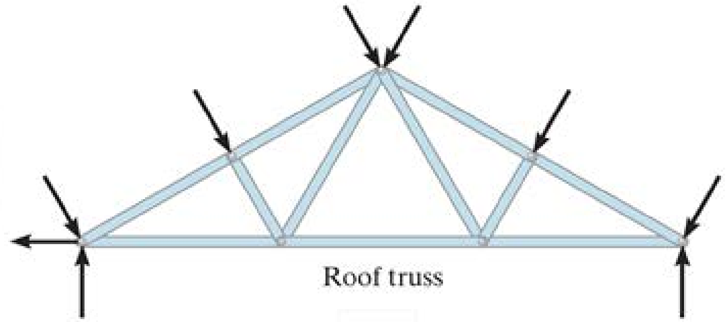
Trusses are commonly used to support roofs.

For a given truss geometry and load, how can you determine the forces in the truss members to be able to select their sizes?

Trusses are also used in a variety of structures like cranes, the frames of aircraft or the space station.

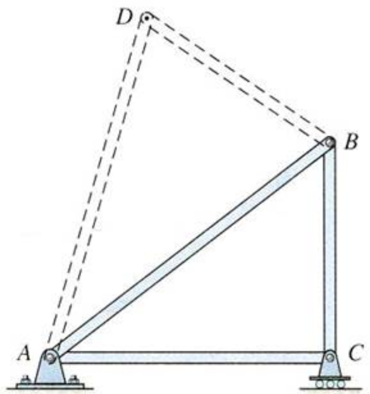
A more challenging question is, that for a given load, how can we design the trusses' geometry to minimize cost?

SIMPLE TRUSSES



A truss is a structure composed of slender members joined together at their end points.

If a truss, along with the imposed load, lies in a single plane (as shown at the top right), then it is called a planar truss.



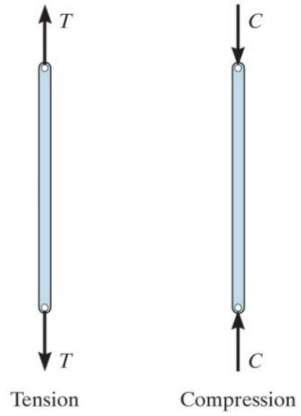
A simple truss is a planar truss which begins with a triangular element and can be expanded by adding two members and a joint. For these trusses, the number of members (M) and the number of joints (J) are related by the equation $M = 2J - 3$.

ANALYSIS & DESIGN ASSUMPTIONS

When designing the members and joints of a truss, first it is necessary to determine the forces in each truss member. This is called the force analysis of a truss. When doing this, two assumptions are made:

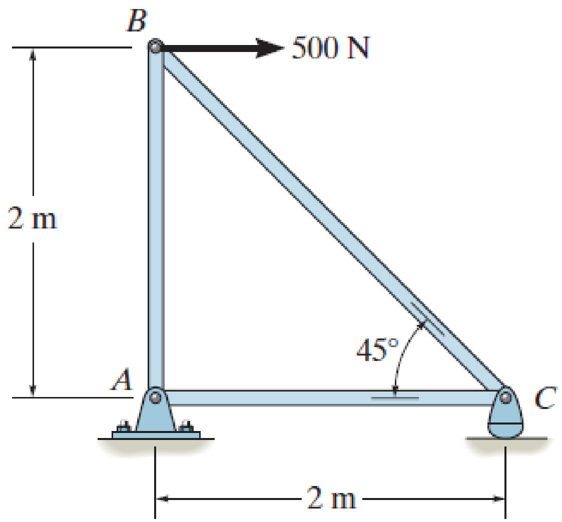
1. All loads are applied at the joints. The weight of the truss members is often neglected as the weight is usually small as compared to the forces supported by the members.

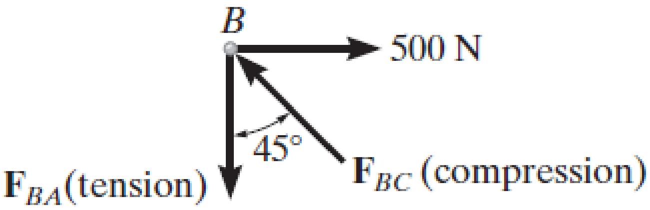
2. The members are joined together by smooth pins. This assumption is satisfied in most practical cases where the joints are formed by bolting the ends together.



With these two assumptions, the members act as two-force members. They are loaded in either tension or compression. Often compressive members are made thicker to prevent "buckling".

THE METHOD OF JOINTS





A free-body diagram of Joint B

When using the method of joints to solve for the forces in truss members, the equilibrium of a joint (pin) is considered. All forces acting at the joint are shown in a FBD. This includes all external forces (including support reactions) as well as the forces acting in the members. Equations of equilibrium ($\sum F_X = 0$ and $\sum F_Y = 0$) are used to solve for the unknown forces acting at the joints.

STEPS FOR ANALYSIS

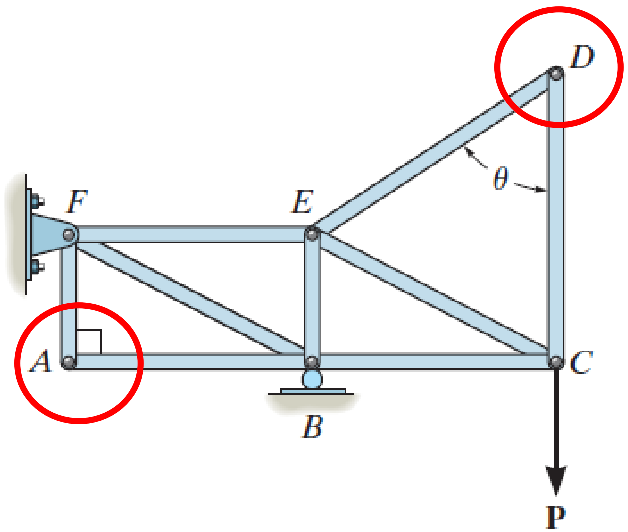
1. If the truss's support reactions are not given, draw a FBD of the entire truss and determine the support reactions (typically using scalar equations of equilibrium).

2. Draw the free-body diagram of a joint with one or two unknowns. Assume that all unknown member forces act in tension (pulling on the pin) unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$, to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

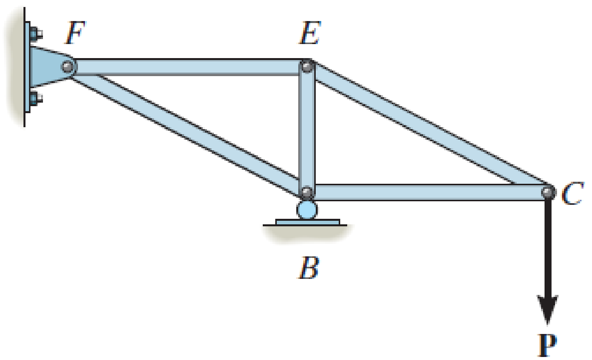
4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.

ZERO-FORCE MEMBERS



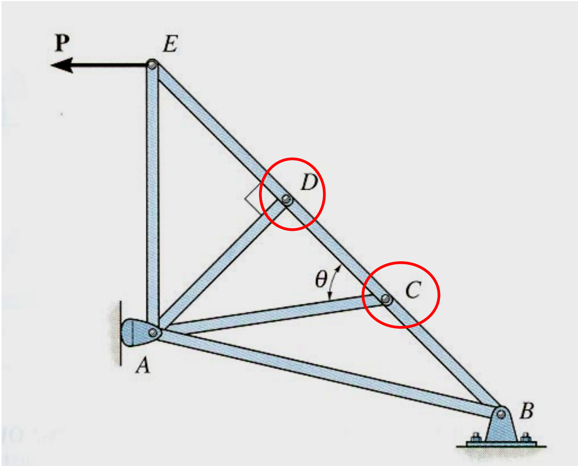
If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-force members. In this example members DE, DC, AF, and AB are zero force members.

You can easily prove these results by applying the equations of equilibrium to joints D and A.



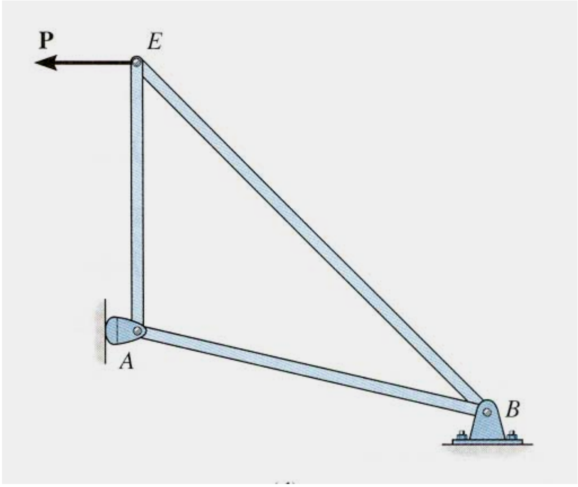
Zero-force members can be removed (as shown in the figure) when analyzing the truss.

ZERO – FORCE MEMBERS (continued)



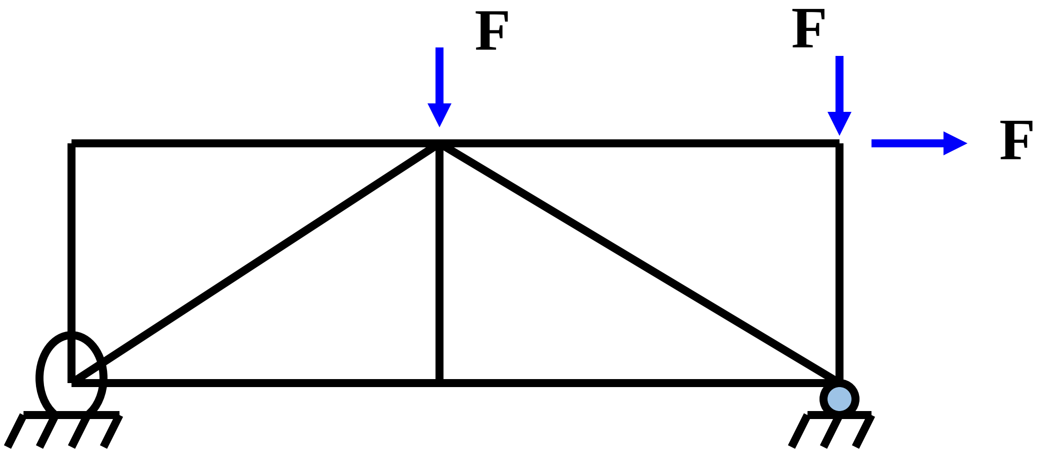
If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member, e.g., DA and CA.

Again, this can easily be proven. One can also remove the zero-force member, as shown, on the left, for analyzing the truss further.



Please note that zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.

QUIZ

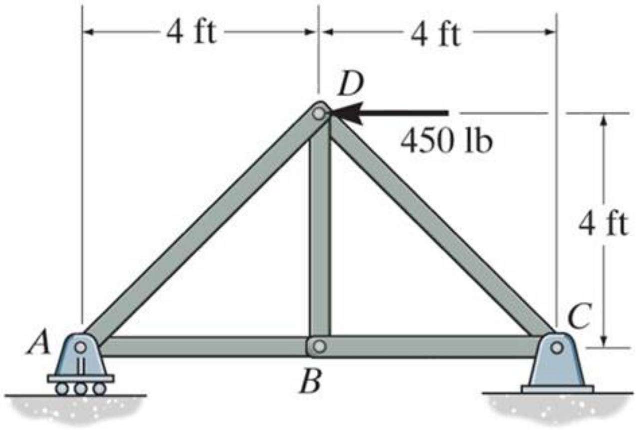


For this truss, determine the number of zero-force members.

A) 0 B) 1 C) 2

D) 3 E) 4

EXAMPLE I



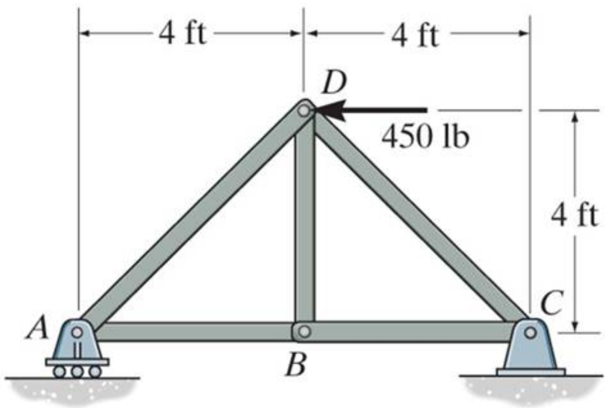
Given: Loads as shown on the truss

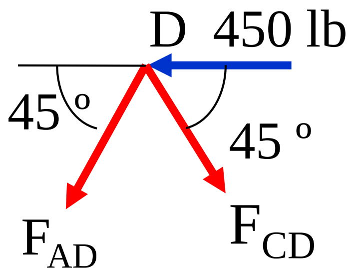
Find: The forces in each member of the truss.

Plan:

1. Check if there are any zero-force members.
2. First analyze pin D and then pin A
3. Note that member BD is zero-force member. $F_{BD} = 0$
4. Why, for this problem, do you not have to find the external reactions before solving the problem?

EXAMPLE I (continued)





FBD of pin D

$$+\rightarrow \sum F_X = -450 + F_{CD}\cos 45° - F_{AD}\cos 45° = 0$$

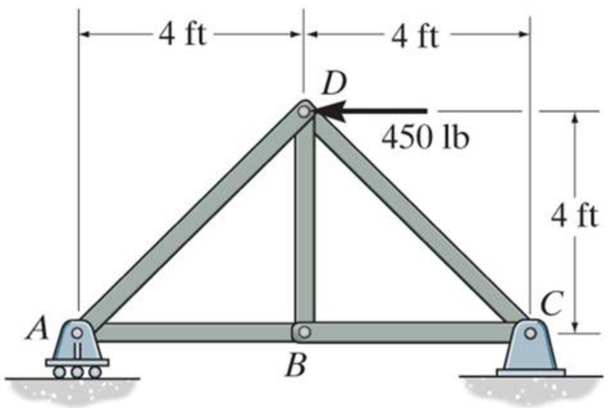
$$+\uparrow \sum F_Y = -F_{CD}\sin 45° - F_{AD}\sin 45° = 0$$

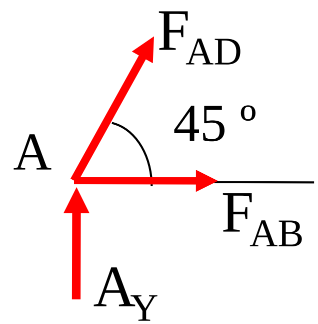
F_{CD} = 318 lb (Tension) or (T)

and F_{AD} = – 318 lb (Compression) or (C)

EXAMPLE I (continued)

Analyzing pin A:





Recall

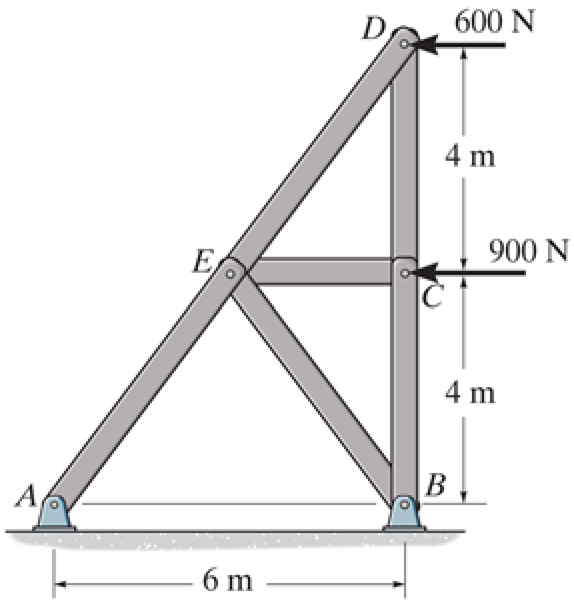
$F_{AD} = -318$ lb

FBD of pin A

$+ \rightarrow \sum F_X = F_{AB} + (-318) \cos 45° = 0;$ $\quad$ $F_{AB} = 225$ lb (T)

Could you have analyzed Joint C instead of A?

EXAMPLE II



Given: Loads as shown on the truss

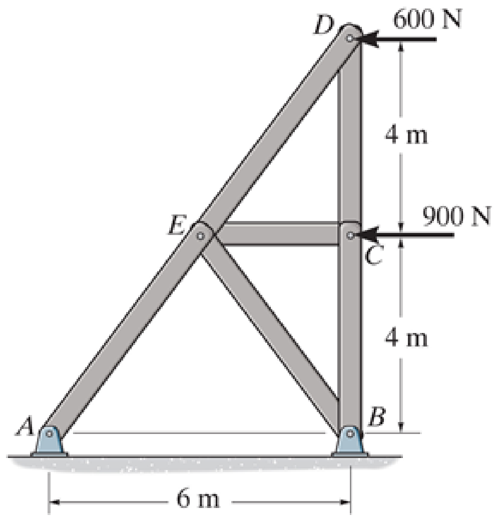
Find: Determine the force in all the truss members (do not forget to mention whether they are in T or C).

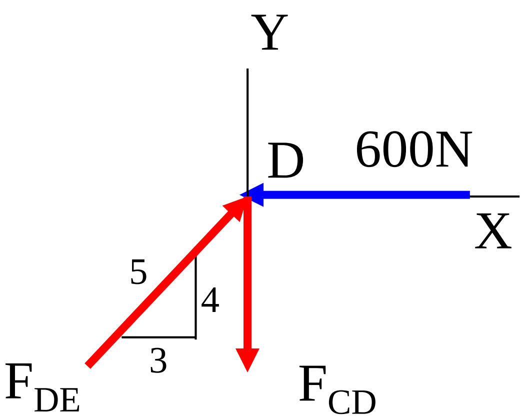
Plan:

a) Check if there are any zero-force members.
Is Member CE zero-force member?

b) Draw FBDs of pins D, C, and E, and then apply E-of-E at those pins to solve for the unknowns.

EXAMPLE II (continued)





Analyzing pin D:

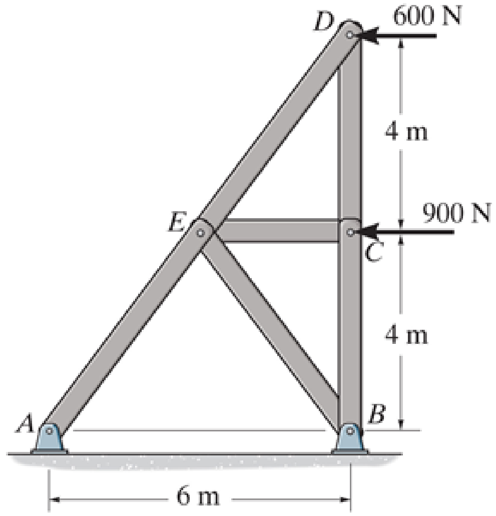
$\rightarrow + \quad \sum F_X = F_{DE}\,(3/5) - 600 = 0$

$F_{DE} = 1000 \text{ N} = \underline{1.00 \text{ kN (C)}}$

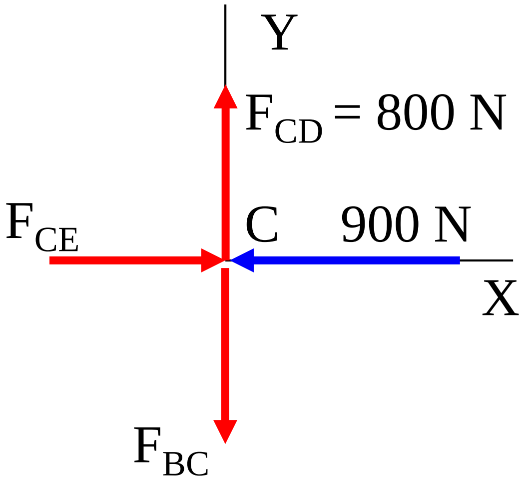
$\uparrow + \quad \sum F_Y = 1000\,(4/5) - F_{CD} = 0$

$F_{CD} = 800 \text{ N} = \underline{0.8 \text{ kN (T)}}$

EXAMPLE II (continued)



FBD of pin C



Analyzing pin C:

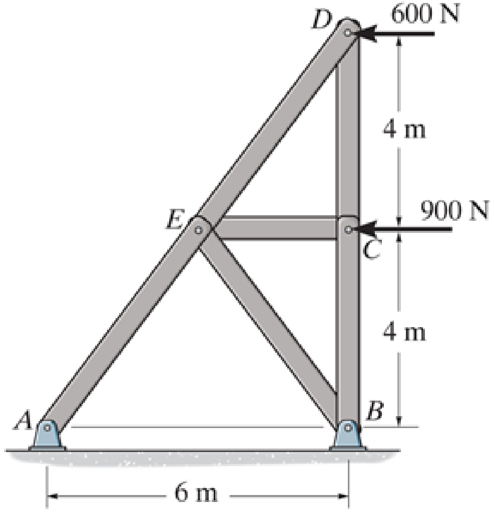
$$\rightarrow + \quad \sum F_X = F_{CE} - 900 = 0$$

$$F_{CE} = 900 \text{ N} = \underline{0.90 \text{ kN (C)}}$$

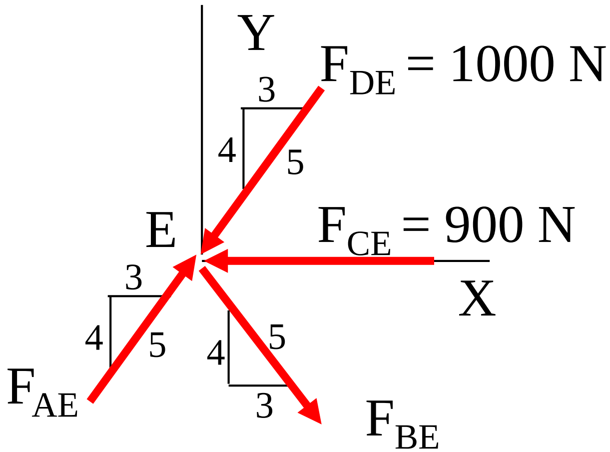
$$\uparrow + \quad \sum F_Y = 800 - F_{BC} = 0$$

$$F_{BC} = 800 \text{ N} = \underline{0.80 \text{ kN (T)}}$$

EXAMPLE II (continued)



FBD of pin E



Analyzing pin E:

$$\rightarrow + \sum F_X = F_{AE}\,(3/5) + F_{BE}\,(3/5) - 1000\,(3/5) - 900 = 0$$

$$\uparrow + \sum F_Y = F_{AE}\,(4/5) - F_{BE}\,(4/5) - 1000\,(4/5) = 0$$

Solving these two equations, we get

F_{AE} = 1750 N = 1.75 kN (C)

F_{BE} = 750 N = 0.75 kN (T)

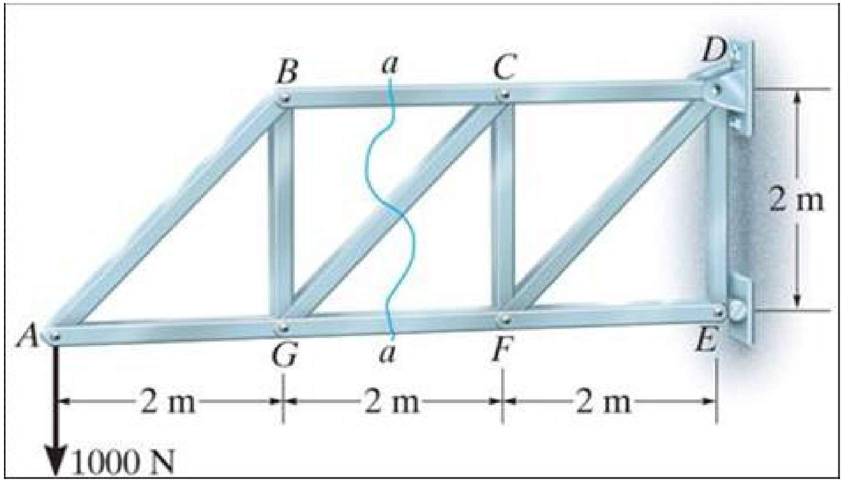
THE METHOD OF SECTIONS

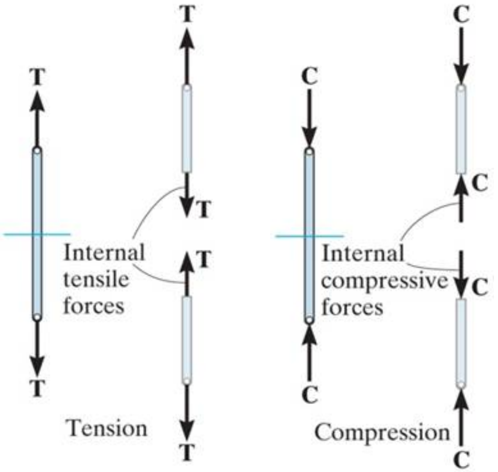
Long trusses are often used to construct large cranes and large electrical transmission towers.

The method of joints requires that many joints be analyzed before we can determine the forces in the middle of a large truss.

So another method to determine those forces is helpful.

THE METHOD OF SECTIONS

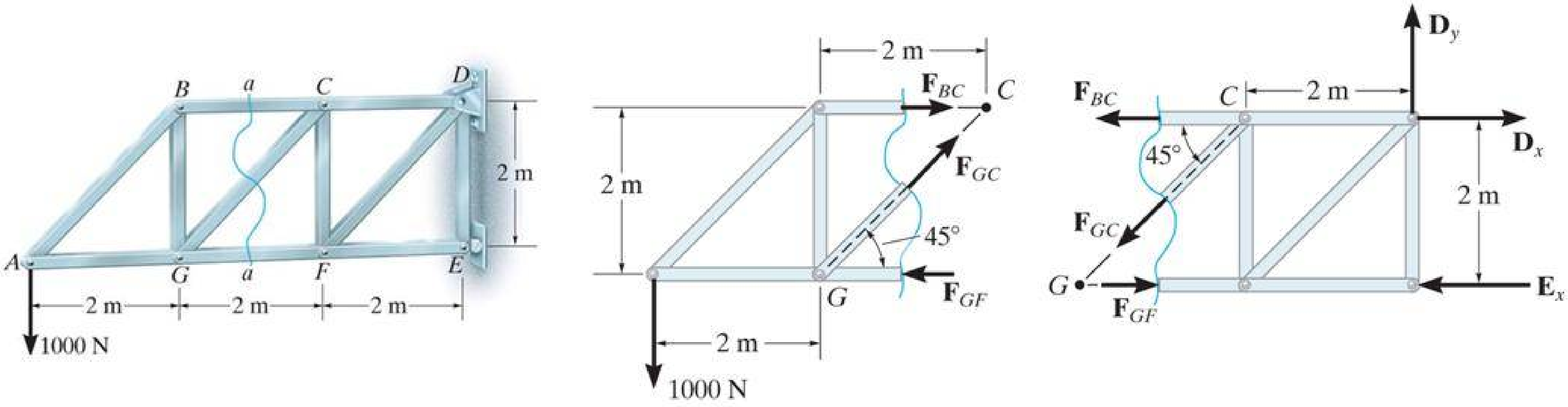




In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.

Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut members also will be either tensile or compressive, with the same magnitude as the forces at the joint. This result is based on the equilibrium principle and Newton's third law.

STEPS FOR ANALYSIS

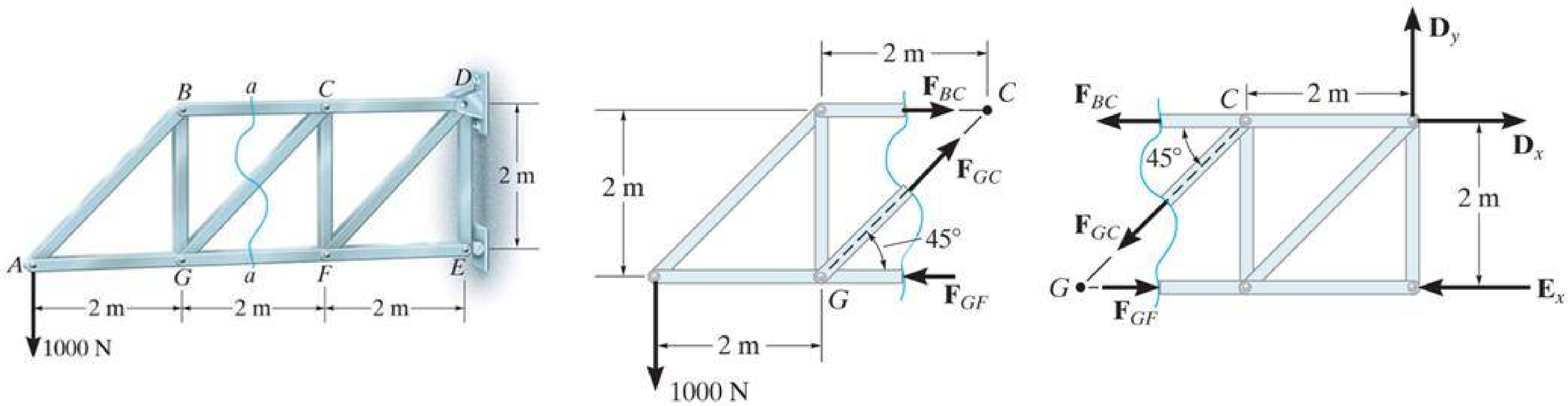


1. Decide how you need to "cut" the truss. This is based on:
   a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).

2. Decide which side of the cut truss will be easier to work with (goal is to minimize the number of external reactions).

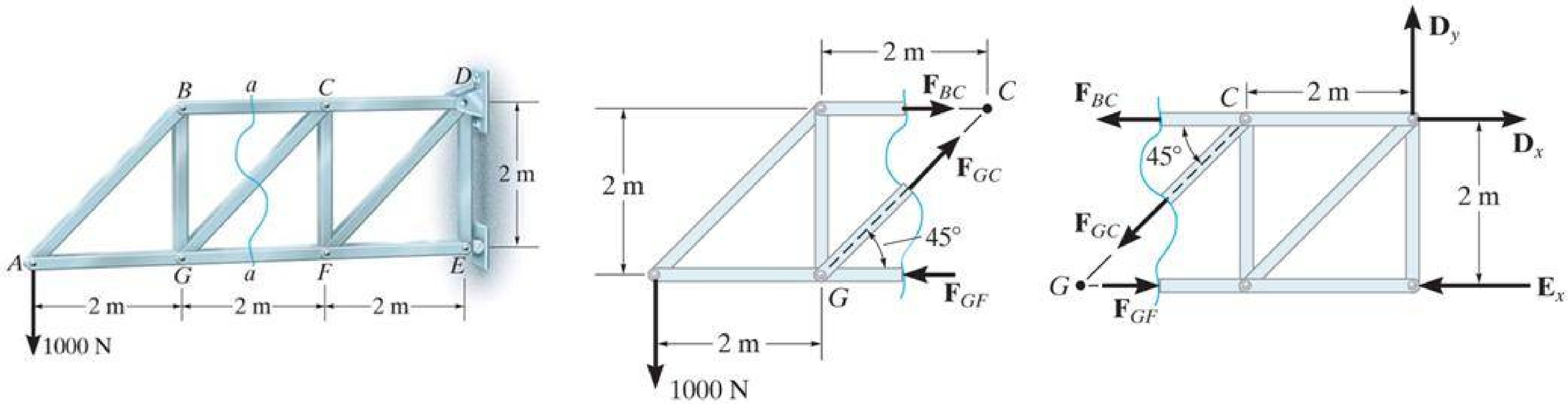
3. If required, determine any necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.

STEPS FOR ANALYSIS (continued)



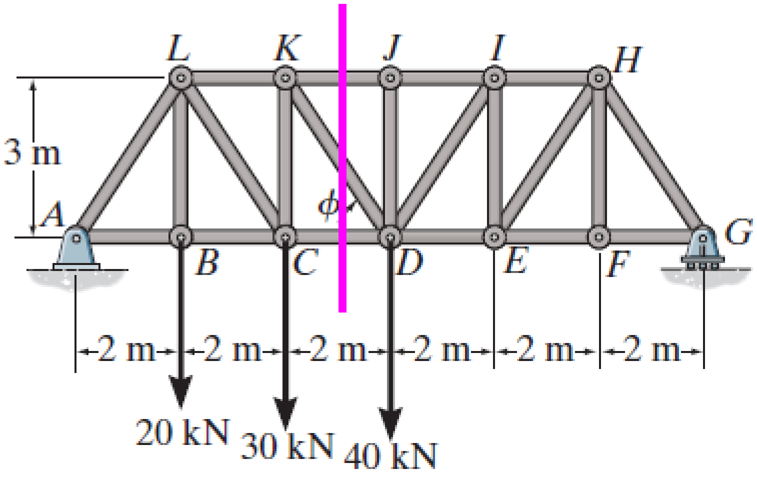
4. Draw the FBD of the selected part of the cut truss. You need to indicate the unknown forces at the cut members. Initially, you may assume all the members are in tension, as done when using the method of joints. Upon solving, if the answer is positive, the member is in tension, as per the assumption. If the answer is negative, the member is in compression. (Please note that you can assume forces to be either tension or compression by inspection as was done in the figures above.)

STEPS FOR ANALYSIS (continued)



5. Apply the scalar equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note, in most cases it is possible to write one equation to solve for one unknown directly. So look for it and take advantage of such a shortcut!

EXAMPLE III



Given: Loads as shown on the truss.

Find: The force in members KJ, KD, and CD.

Plan:

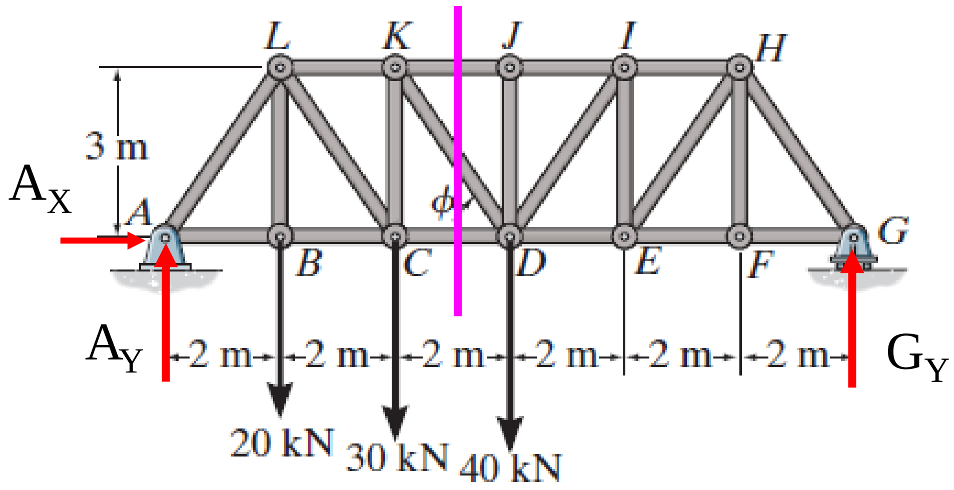
a) Take a cut through members KJ, KD and CD.

b) Work with the left piece of the cut sections.

c) Determine the support reactions at A. What are they?

d) Apply the E-of-E to find the forces in KJ, KD and CD.

EXAMPLE III (continued)



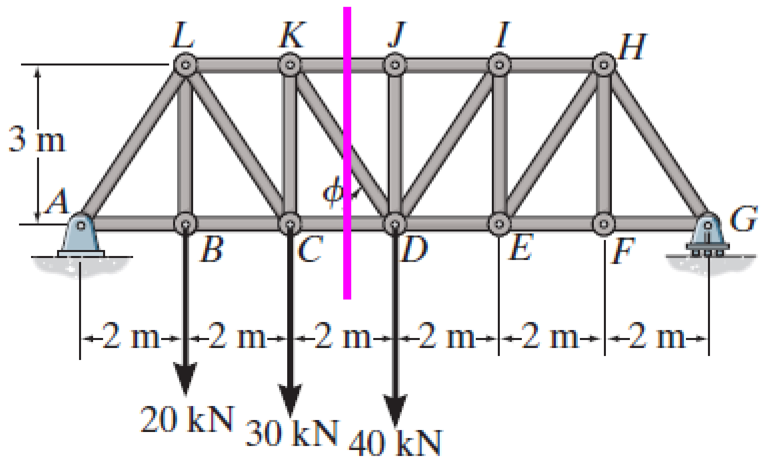
Analyzing the entire truss for the reactions at A, we get

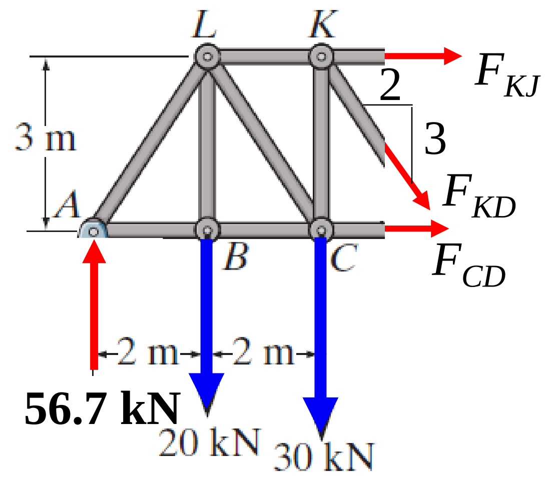
$\Sigma F_X = A_X = 0.$

A moment equation about G to find A_Y results in:

$\Sigma M_G = -A_Y (12) + 20 (10) + 30 (8) + 40 (6) = 0;$ $\underline{A_Y = 56.7 \text{ kN}}$

EXAMPLE III (continued)



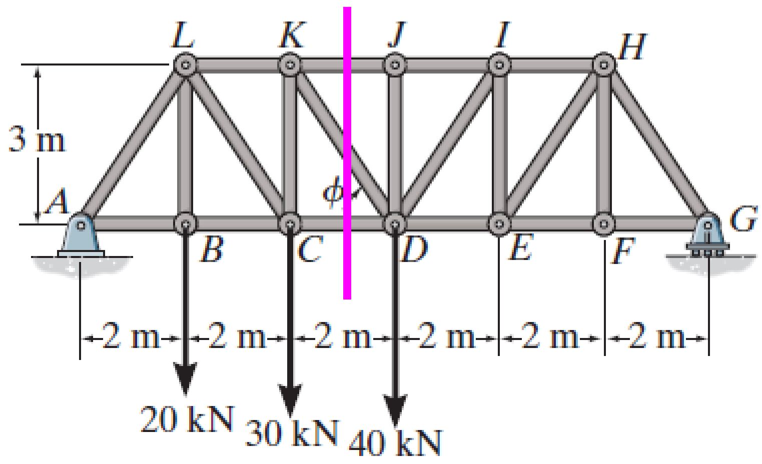


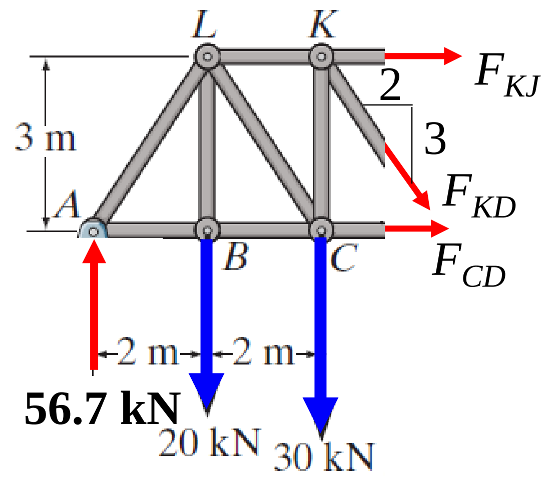
Now take moments about point D. Why do this?

$$+ M_D = -56.7\ (6) + 20\ (4) + 30\ (2) - F_{KJ}\ (3) = 0$$

$F_{KJ} = -66.7$ kN or 66.7 kN (C)

EXAMPLE III (continued)





Now use the x and y-directions equations of equilibrium.

$$\uparrow + \Sigma F_Y = 56.7 - 20 - 30 - (3/\sqrt{13})\, F_{KD} = 0;$$

$\underline{F_{KD} = 8.05 \text{ kN (T)}}$

$$\rightarrow + \Sigma F_X = (-66.7) + (2/\sqrt{13})\,(8.05) + F_{CD} = 0;$$

$\underline{F_{CD} = 62.2 \text{ kN (T)}}$

CONCEPT QUIZ

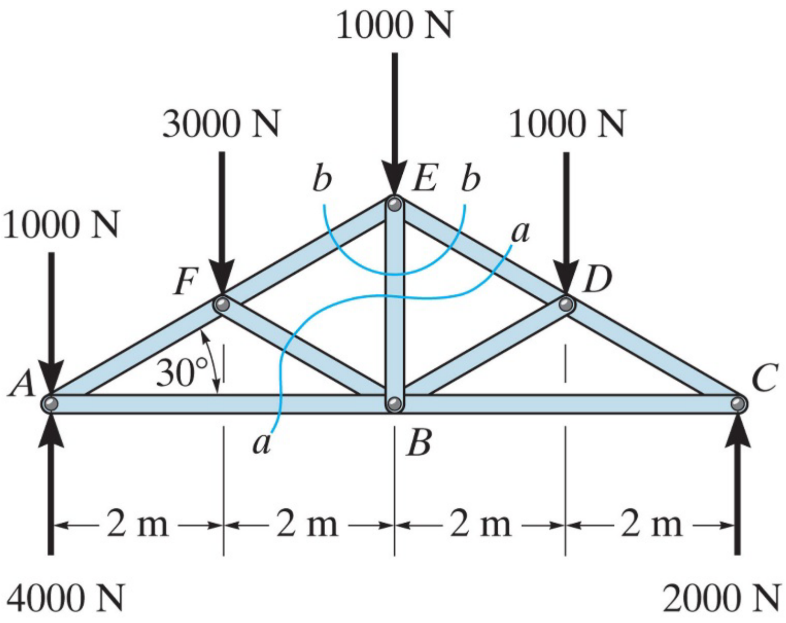
1. Can you determine the force in member ED by making the cut at section a-a? Explain your answer.

   A) No, there are four unknowns.

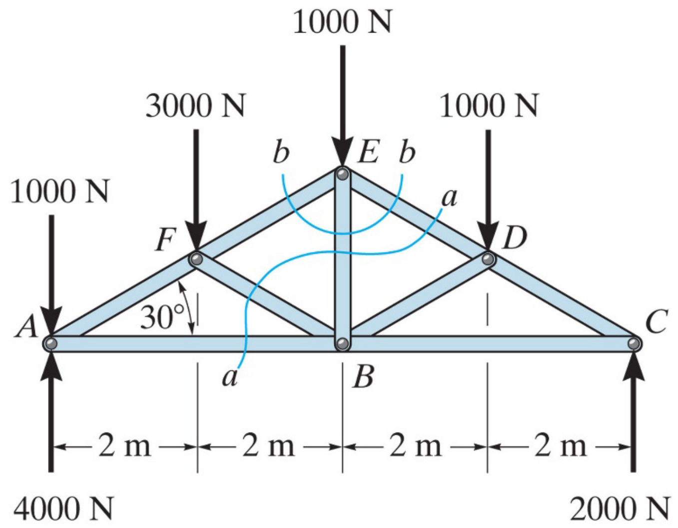
   B) Yes, using $\Sigma M_D = 0$.

   C) Yes, using $\Sigma M_E = 0$.

   D) Yes, using $\Sigma M_B = 0$.



CONCEPT QUIZ (continued)



2. If you know F_{ED}, how will you determine F_{EB}?

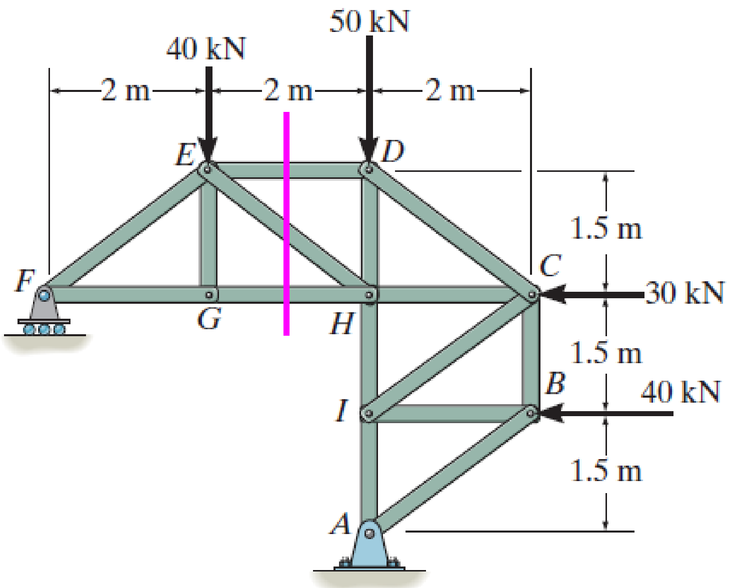
A) By taking section b-b and using $\Sigma\ M_E = 0$

B) By taking section b-b, and using $\Sigma\ F_X = 0$ and $\Sigma\ F_Y = 0$

C) By taking section a-a and using $\Sigma\ M_B = 0$

D) By taking section a-a and using $\Sigma\ M_D = 0$

EXAMPLE IV



Given: Loads as shown on the truss.

Find: The forces in members ED, EH, and GH.

Plan:

a) Take the cut through members ED, EH, and GH.

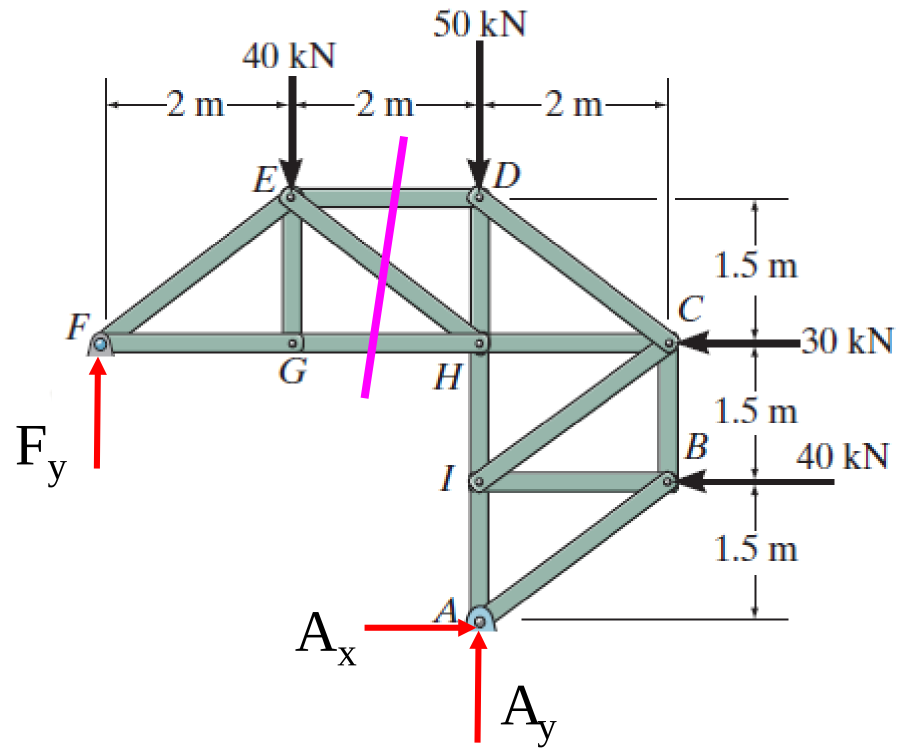
b) Analyze the left section. Determine the support reactions at F. [Why?]

c) Draw the FBD of the left section.

d) Apply the equations of equilibrium (if possible, try to do it so that every equation yields an answer to one unknown.

EXAMPLE IV (continued)

1) Determine the support reactions at F by drawing the FBD of the entire truss.

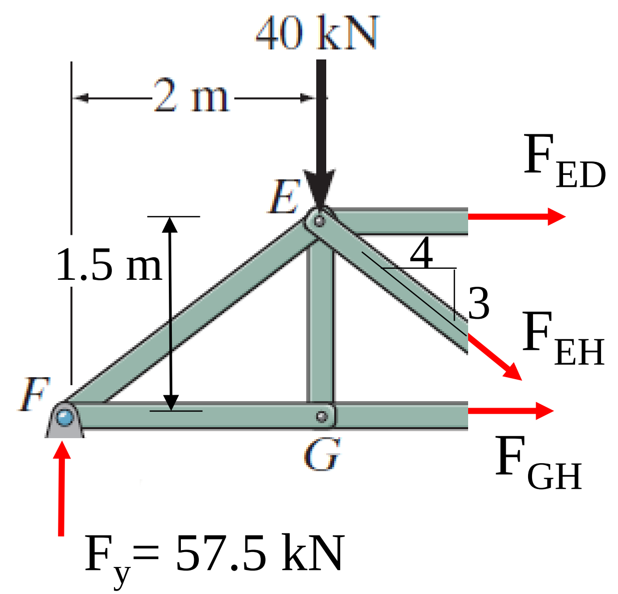


$$\curvearrowleft + \Sigma M_A = -F_y\,(4) + 40\,(2) + 30\,(3) + 40\,(1.5) = 0;$$

$\underline{F_y = 57.5\text{ kN}}$

EXAMPLE IV (continued)

2) Analyze the left section.



$\curvearrowleft + \Sigma M_E = -57.5\,(2) + F_{GH}\,(1.5) = 0;$

$\underline{F_{GH} = 76.7 \text{ kN (T)}}$

$\uparrow + \Sigma F_y = 57.5 - 40 - F_{EH}\,(3/5) = 0;$

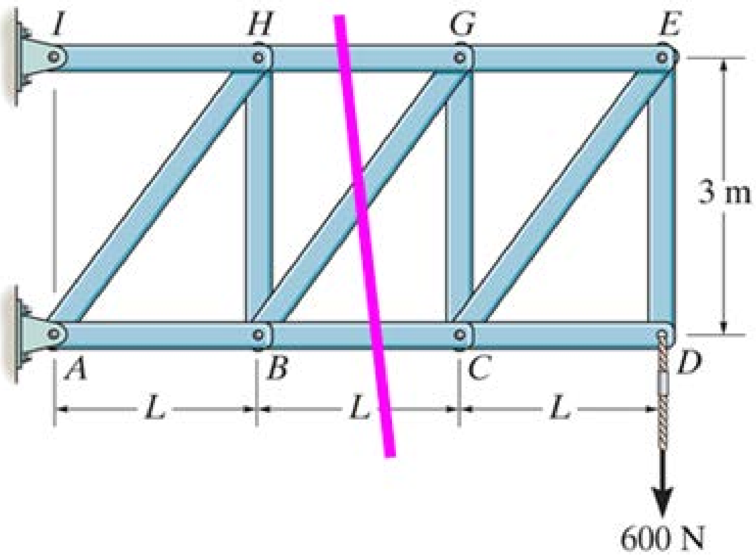
$\underline{F_{EH} = 29.2 \text{ kN (T)}}$

$\curvearrowleft + \Sigma M_H = -57.5\,(4) + 40\,(2) - F_{ED}\,(1.5) = 0;$

$F_{ED} = -100 \text{ kN} = \underline{100 \text{ kN (C)}}$

ATTENTION QUIZ

1. As shown, a cut is made through members GH, BG and BC to determine the forces in them. Which section will you choose for analysis and why?



   A) Right, fewer calculations.

   B) Left, fewer calculations.

   C) Either right or left, same amount of work.

   D) None of the above, too many unknowns.

ATTENTION QUIZ

2. When determining the force in member HG in the previous question, which one equation of equilibrium is the best one to use?

A) $\Sigma M_H = 0$

B) $\Sigma M_G = 0$

C) $\Sigma M_B = 0$

D) $\Sigma M_C = 0$