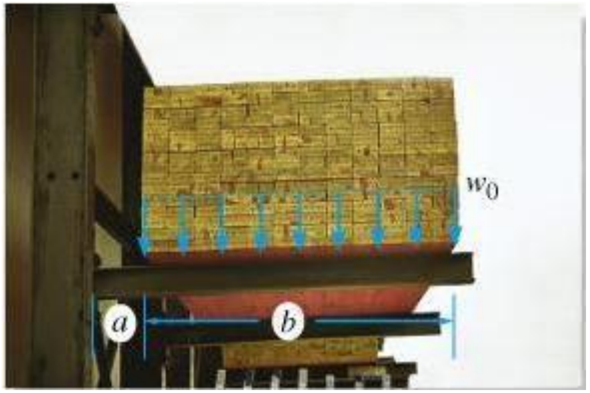
REDUCTION OF A SIMPLE DISTRIBUTED LOADING

Objectives:

Determine an equivalent force for a distributed load.



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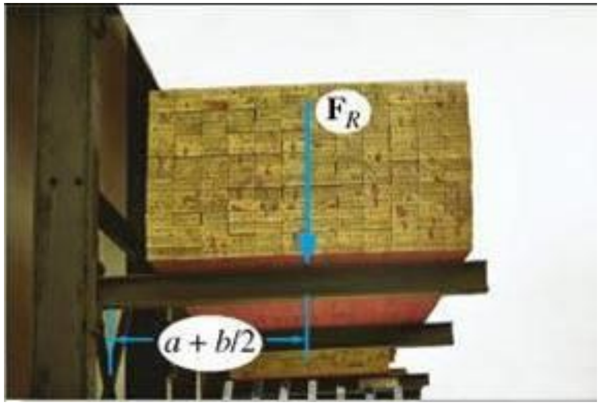
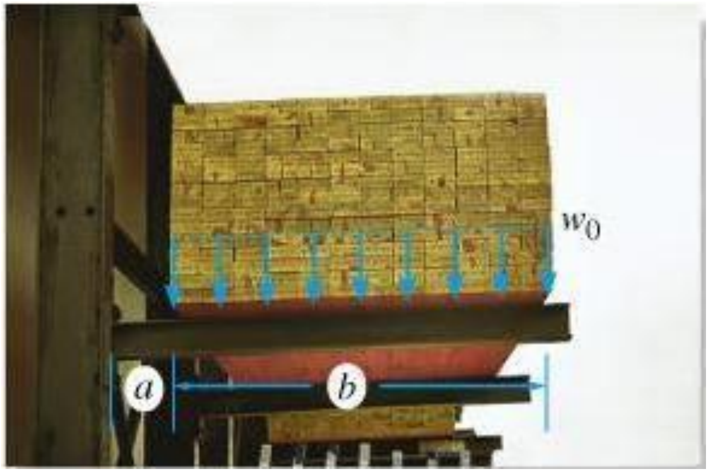


APPLICATIONS



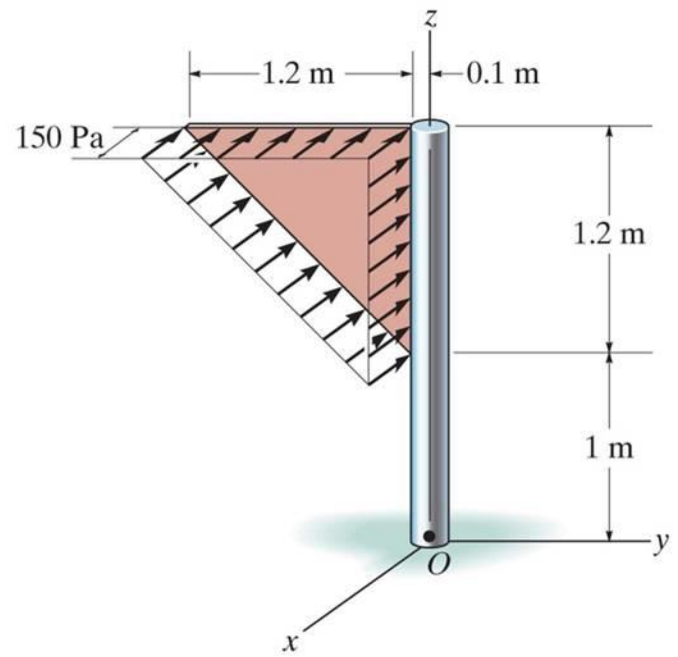
There is a bundle (called a bunk) of 2” x 4” boards stored on a storage rack. This lumber places a distributed load (due to the weight of the wood) on the beams holding the bunk.

To analyze the load’s effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

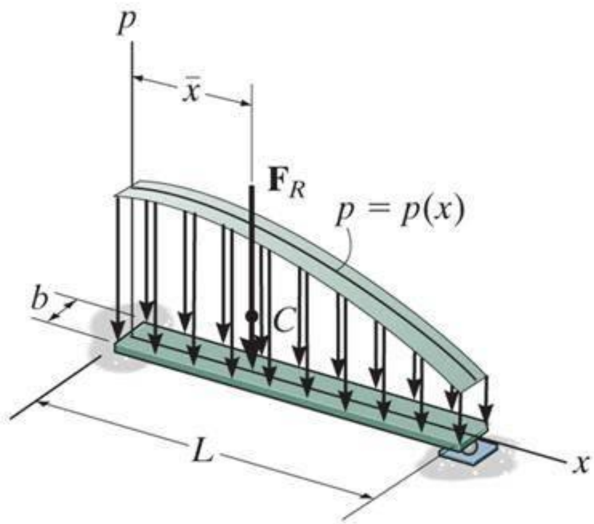
APPLICATIONS (continued)

The uniform wind pressure is acting on a triangular sign (shown in light brown).

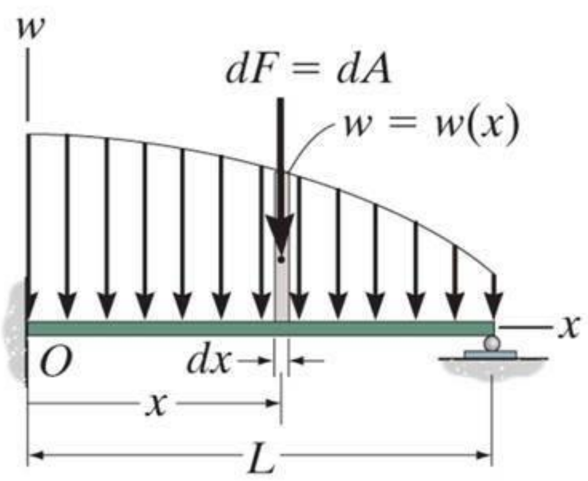
To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.



DISTRIBUTED LOADING

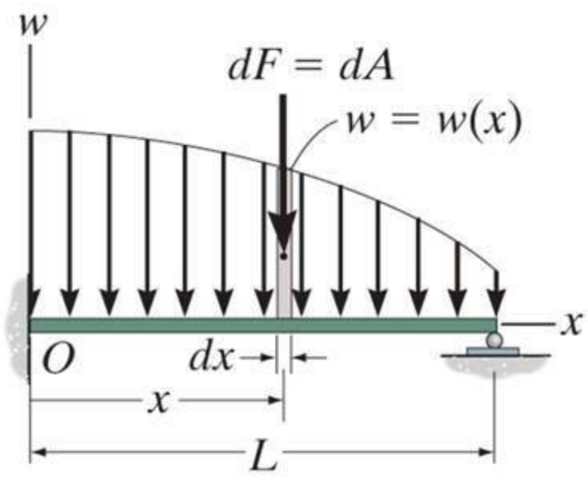


In many situations, a surface area of a body is subjected to a distributed load. Such forces are caused by winds, fluids, or the weight of items on the body's surface.



In such cases, the distributed load, w, is a function of x and has units of force per length.

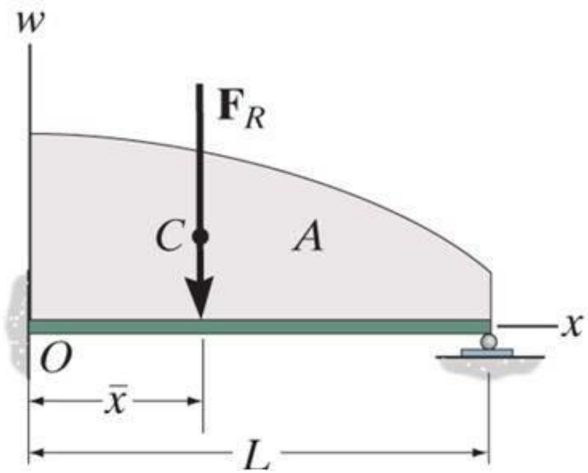
MAGNITUDE OF RESULTANT FORCE



Consider an element of length dx.

The force magnitude dF acting on it is given as

$$dF = w(x)\,dx$$

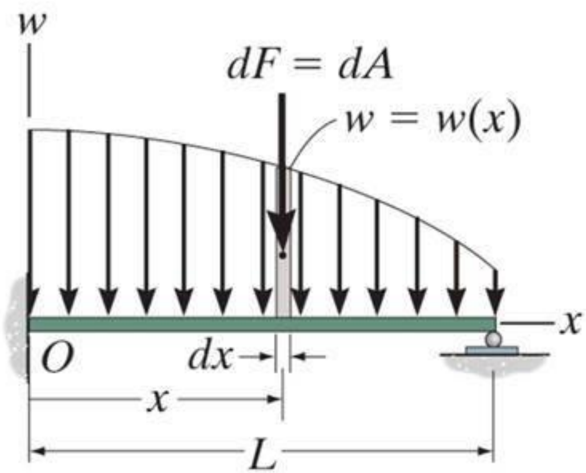


The net force on the beam is given by

$$+\downarrow F_R = \int_L dF = \int_L w(x)\,dx = A$$

Here A is the area under the loading curve w(x).

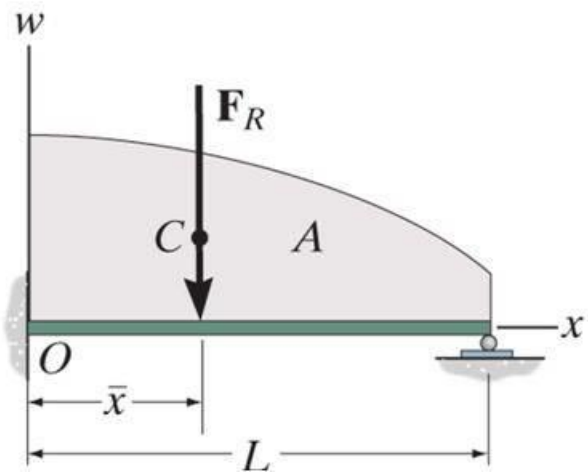
LOCATION OF THE RESULTANT FORCE



The force dF will produce a moment of (x)(dF) about point O.

The total moment about point O is given as

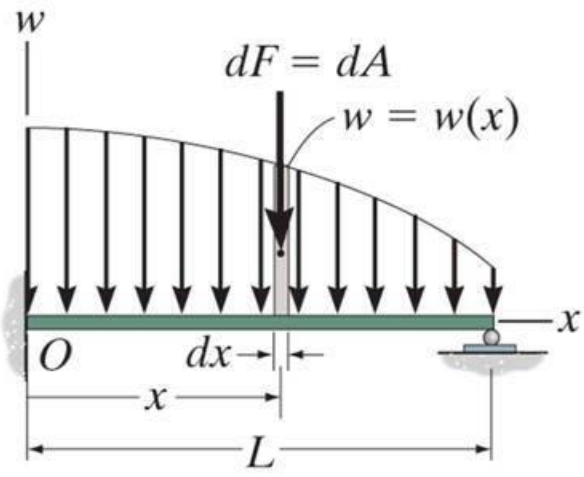
$$\circlearrowleft + \; M_{RO} = \int_L x \, dF = \int_L x \, w(x) \, dx$$



Assuming that F_R acts at $\bar{x}$, it will produce the same moment about point O as

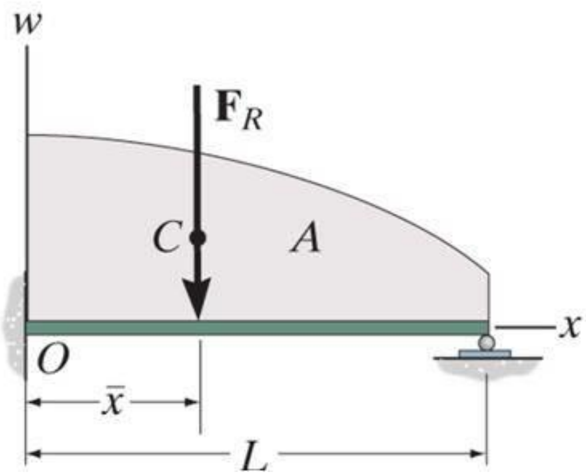
$$\circlearrowleft + \; M_{RO} = (\bar{x})\,(F_R) = \bar{x} \int_L w(x) \, dx$$

LOCATION OF THE RESULTANT FORCE (continued)



Comparing the last two equations, we get

$$\bar{x} = \frac{\int_L x w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



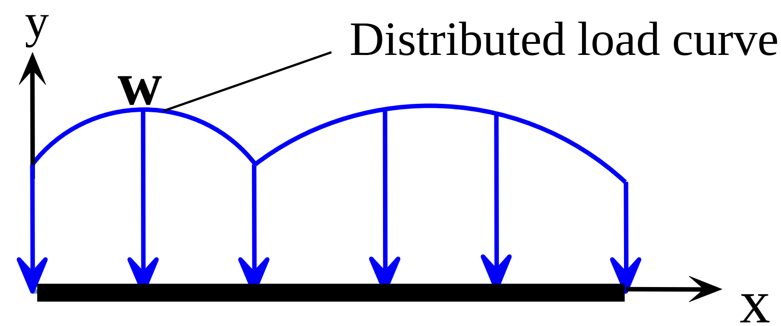
F_R acts through a point “C,” which is the geometric center or centroid of the area under the loading curve w(x).

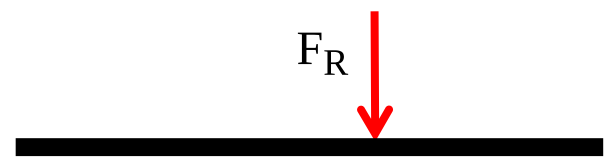
QUIZ

1. The resultant force (F_R) due to a distributed load is equivalent to the _____ under the distributed loading curve, w = w(x).

   A) Centroid B) Arc length

   C) Area D) Volume





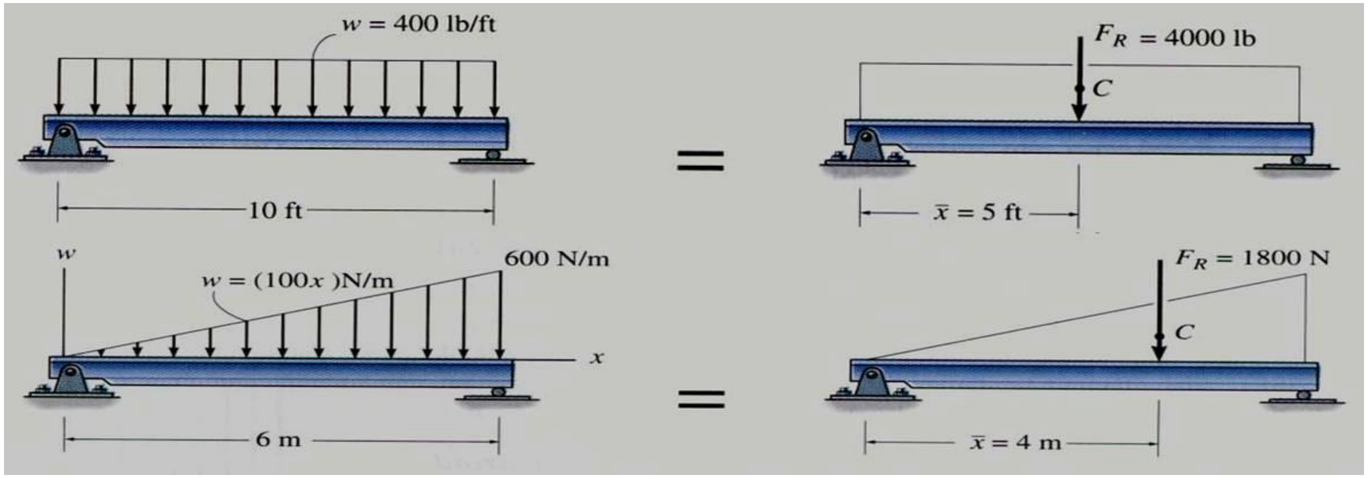
2. The line of action of the distributed load's equivalent force passes through the ______ of the distributed load.

   A) Centroid B) Mid-point

   C) Left edge D) Right edge

EXAMPLE I

Find the equivalent concentrated loads (which is a common name for the resultant of the distributed load).



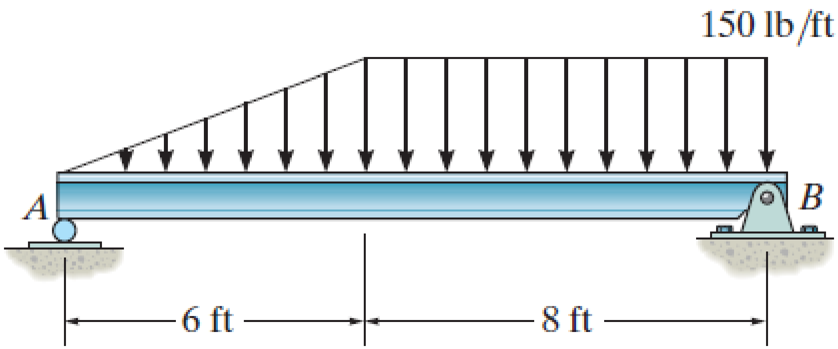
The rectangular load: $F_R = 400 \times 10 =$ 4,000 lb and $\bar{x} =$ 5 ft.

The triangular loading:

$F_R = (0.5)\,(600)\,(6) =$ 1,800 N and $\bar{x} = 6 - (1/3)\,6 =$ 4 m.

Please note that the centroid of a right triangle is at a distance one third the width of the triangle as measured from its base.

EXAMPLE II



Given: The loading on the beam as shown.

Find: The equivalent force and its location from point A.

Plan:

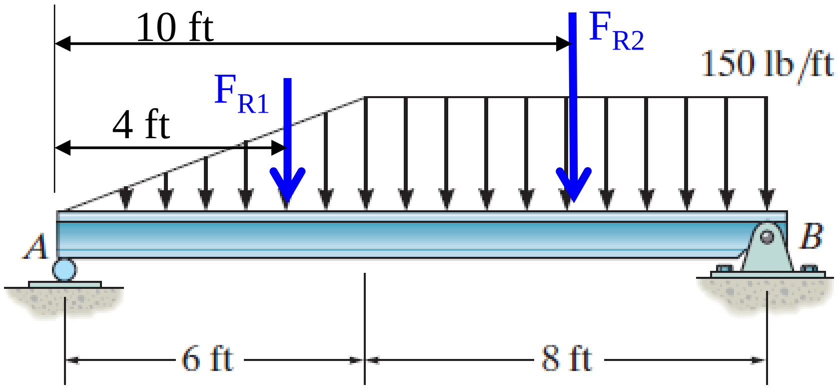
1) The distributed loading can be divided into two parts. (one rectangular loading and one triangular loading).

2) Find F_R and its location for each of the distributed loads.

3) Determine the overall F_R of the point loadings and its location.

NOTE: You can use the composite body method in Chapter 9, as well.

EXAMPLE II (continued)



For the triangular loading of height 150 lb/ft and width 6 ft,

F_{R1} = (0.5) (150) (6) = 450 lb

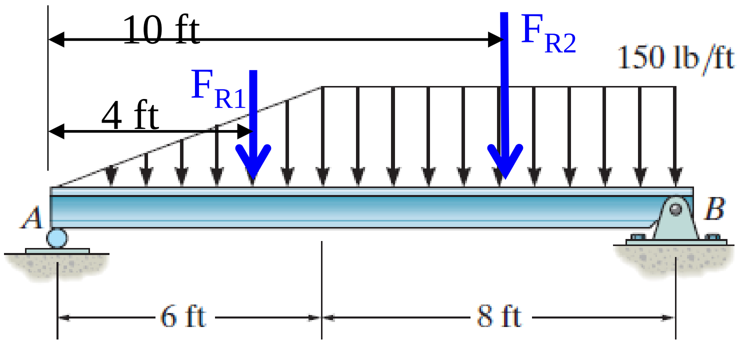
and its line of action is at $\bar{x}_1$ = (2/3)(6) = 4 ft from A

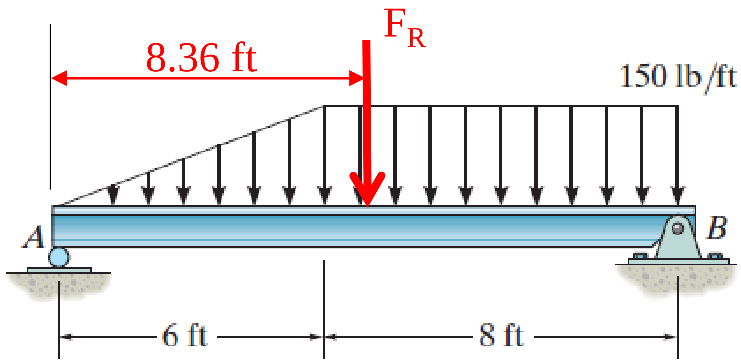
For the rectangular loading of height 150 lb/ft and width 8 ft,

F_{R2} = (150) (8) = 1200 lb

and its line of action is at $\bar{x}_2$= 6 + (1/2)(8) = 10 ft from A

EXAMPLE II (continued)





The equivalent force and couple moment at A will be

$$F_R = 450 + 1200 = \underline{1650 \text{ lb}}$$

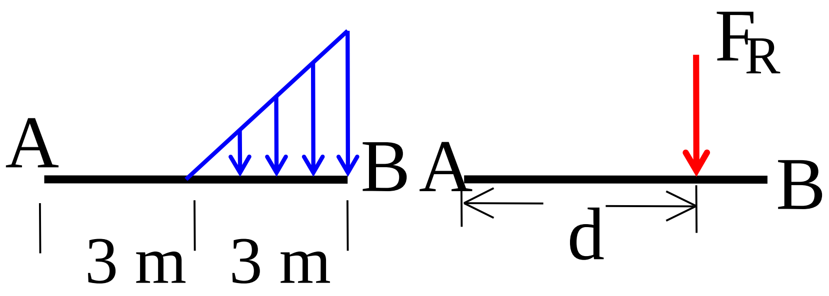
$$+\circlearrowleft M_{RA} = 4\,(450) + 10(1200) = \underline{13800 \text{ lb}\cdot\text{ft}}$$

Since ($F_R\ \bar{x}$) has to equal M_{RA} : $1650\ \bar{x} = 13800$

Solve for $\bar{x}$ to find the equivalent force's location.

$\bar{x}$ = 8.36 ft from A.

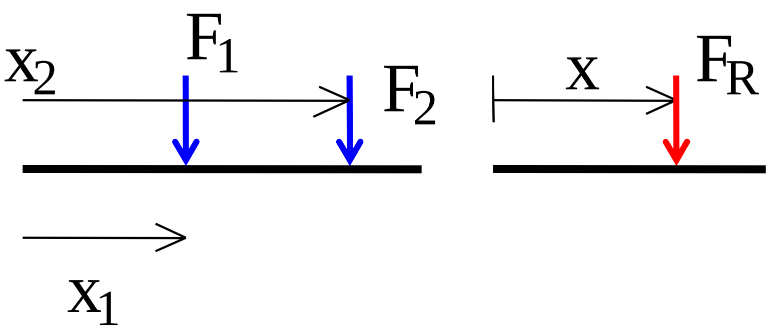
CONCEPT QUIZ



1. What is the location of F_R, i.e., the distance d?

A) 2 m B) 3 m C) 4 m

D) 5 m E) 6 m

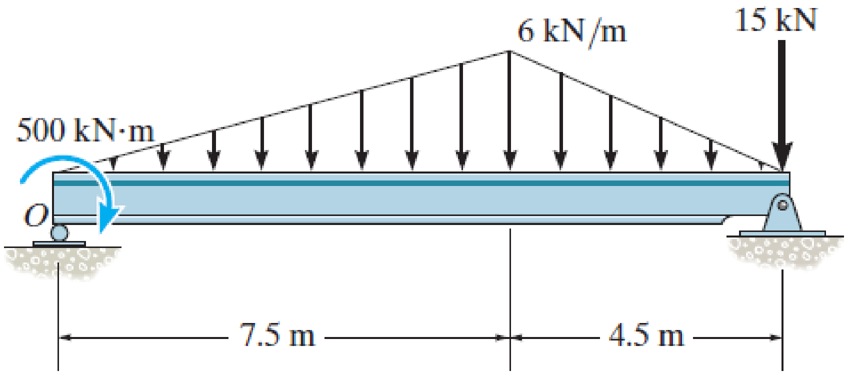


2. If F_1 = 1 N, x_1 = 1 m, F_2 = 2 N and x_2 = 2 m, what is the location of F_R, i.e., the distance x.

A) 1 m B) 1.33 m C) 1.5 m

D) 1.67 m E) 2 m

EXAMPLE III



Given: The distributed loading on the beam as shown.

Find: The equivalent force and couple moment acting at point O.

Plan:

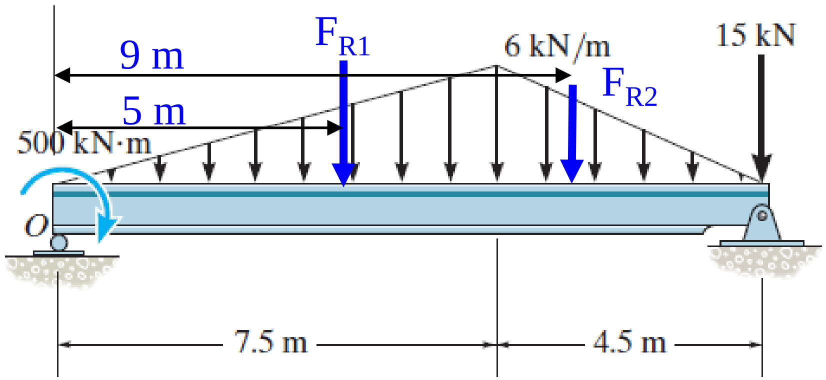
1) The distributed loading can be divided into two parts--two triangular loads.

2) Find F_R and its location for each of these distributed loads.

3) Determine the overall F_R of the point loadings and couple moment at point O.

NOTE: You can use the composite body method in Chapter 9 as well.

EXAMPLE III (continued)



For the left triangular loading of height 6 kN/m and width 7.5 m,

F_{R1} = (0.5) (6) (7.5) = 22.5 kN

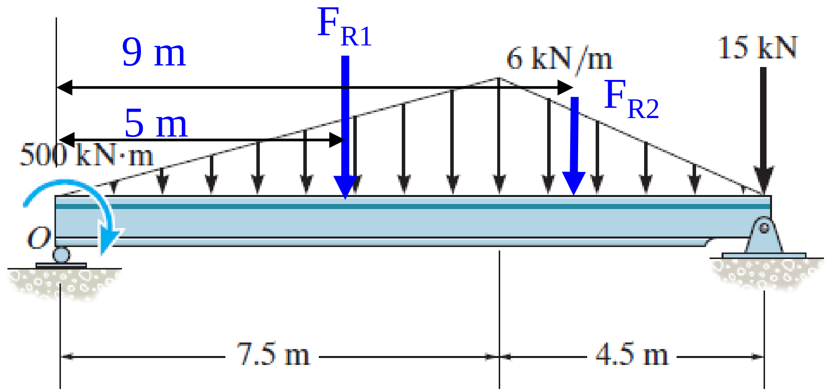
and its line of action is at $\bar{x}_1$ = (2/3)(7.5) = 5 m from O

For the right triangular loading of height 6 kN/m and width 4.5 m,

F_{R2} = (0.5) (6) (4.5) = 13.5 kN

and its line of action is at $\bar{x}_2$ = 7.5 + (1/3)(4.5) = 9 m from O

EXAMPLE III (continued)



For the combined loading of the three forces, add them.

$$F_R = 22.5 + 13.5 + 15 = 51 \text{ kN}$$

The couple moment at point O will be

$$+\circlearrowleft M_{RO} = 500 + 5\,(22.5) + 9\,(13.5) + 12\,(15) = 914 \text{ kN}\cdot\text{m}$$