POSITION VECTOR (APPLICATION)

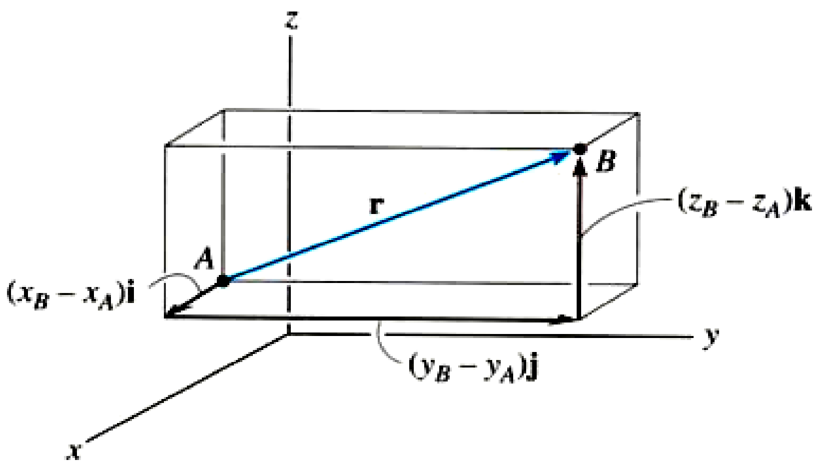
This ship's mooring line, can be represented as a Cartesian vector.

What are the forces in the mooring line and how do we find their directions?

Why would we want to know these things?

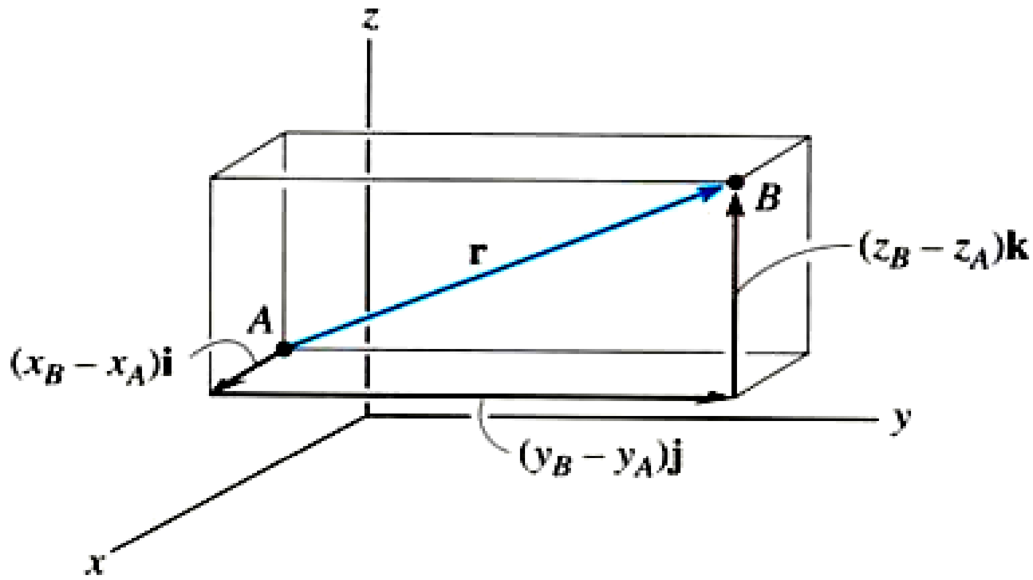
POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A and B, in 3-D space.
Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

POSITION VECTOR (continued)

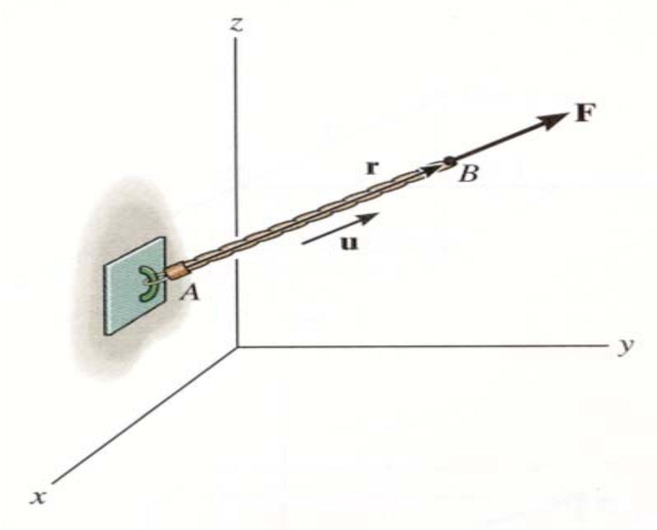


The position vector directed from A to B, $\boldsymbol{r_{AB}}$, is defined as

$$\boldsymbol{r_{AB}} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point. ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE
(Section 2.8)



If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude. So we need to:

a) Find the position vector, $\mathbf{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\mathbf{u}_{AB} = (\mathbf{r}_{AB}/r_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\mathbf{F} = F\ \mathbf{u}_{AB}$.

QUIZ

1. The position vector $\boldsymbol{r}_{PQ}$ is obtained by

   A) Coordinates of Q minus coordinates of the origin

   B) Coordinates of P minus coordinates of Q

   C) Coordinates of Q minus coordinates of P

   D) Coordinates of the origin minus coordinates of P

2. A force of magnitude F, directed along a unit vector $\boldsymbol{U}$, is given by $\boldsymbol{F}$ = ______ .

   A) F ($\boldsymbol{U}$)

   B) $\boldsymbol{U}$ / F

   C) F / $\boldsymbol{U}$

   D) F + $\boldsymbol{U}$

   E) F – $\boldsymbol{U}$

QUIZ

3. **P** and **Q** are two points in a 3-D space. How are the position vectors $\mathbf{r}_{PQ}$ and $\mathbf{r}_{QP}$ related?

   A) $\mathbf{r}_{PQ} = \mathbf{r}_{QP}$ B) $\mathbf{r}_{PQ} = -\mathbf{r}_{QP}$

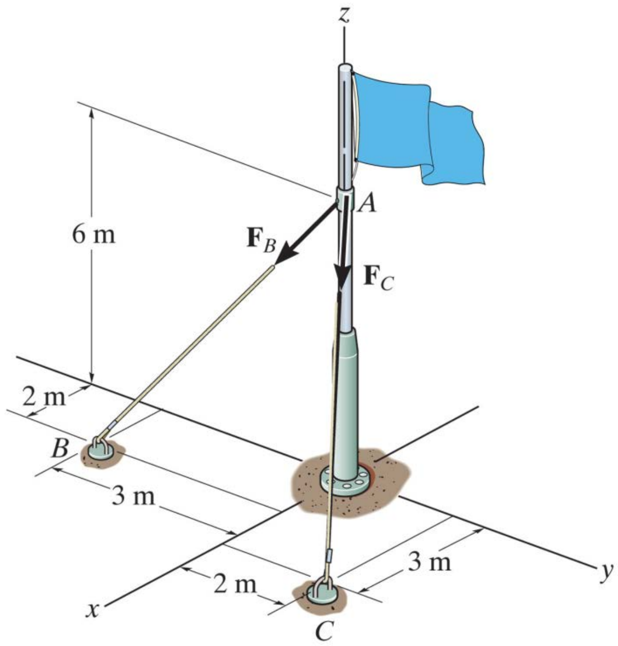
   C) $\mathbf{r}_{PQ} = 1/\mathbf{r}_{QP}$ D) $\mathbf{r}_{PQ} = 2\,\mathbf{r}_{QP}$

4. A force vector, $\mathbf{F}$, directed along a line defined by PQ is given by

   A) $(\mathbf{F}/F)\,\mathbf{r}_{PQ}$ B) $\mathbf{r}_{PQ}/r_{PQ}$

   C) $F(\mathbf{r}_{PQ}/r_{PQ})$ D) $F(r_{PQ}/\mathbf{r}_{PQ})$

Example VII



Given: Two forces are acting on a flag pole as shown in the figure. $F_B = 560$ N and $F_C = 700$ N

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Find the forces along AB and AC in the Cartesian vector form.

2) Add the two forces to get the resultant force, $\boldsymbol{F_R}$.

3) Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

Example VII (continued)

$\boldsymbol{r_{AB}} = \{2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$\boldsymbol{r_{AC}} = \{3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$r_{AB} = \{2^2 + (-3)^2 + (-6)^2\}^{1/2} = 7$ m

$r_{AC} = \{3^2 + 2^2 + (-6)^2\}^{1/2} = 7$ m

$\boldsymbol{F_{AB}} = 560\,(\boldsymbol{r_{AB}} / r_{AB})$ N

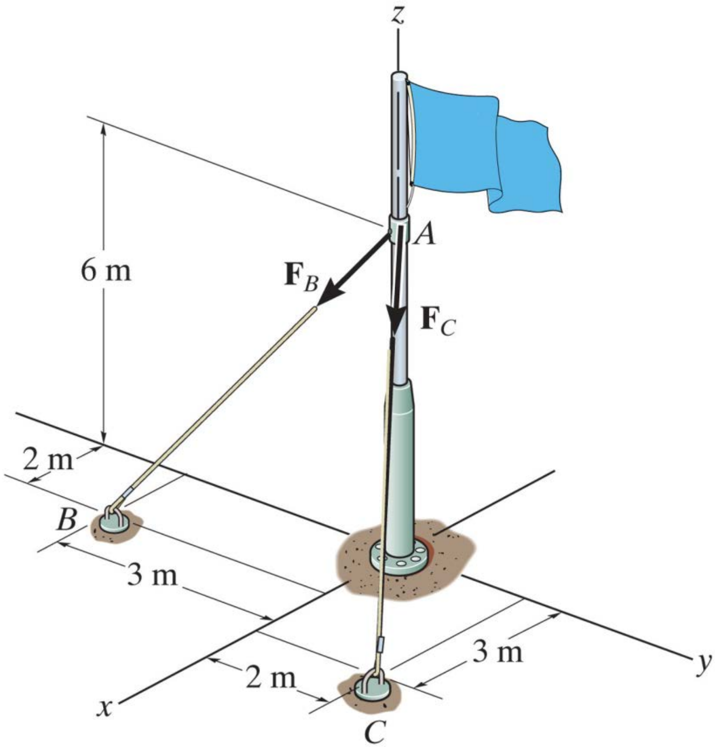
$\boldsymbol{F_{AB}} = 560\,(2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F_{AB}} = (160\,\boldsymbol{i} - 240\,\boldsymbol{j} - 480\,\boldsymbol{k})$ N

$\boldsymbol{F_{AC}} = 700\,(\boldsymbol{r_{AC}} / r_{AC})$ N

$\boldsymbol{F_{AC}} = 700\,(3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F_{AC}} = \{300\,\boldsymbol{i} + 200\,\boldsymbol{j} - 600\,\boldsymbol{k}\}$ N



Example VII (continued)

$\boldsymbol{F_R} = \boldsymbol{F_{AB}} + \boldsymbol{F_{AC}}$

$= \{460\ \boldsymbol{i} - 40\ \boldsymbol{j} - 1080\ \boldsymbol{k}\}$ N

$F_R = \{460^2 + (-40)^2 + (-1080)^2\}^{1/2}$

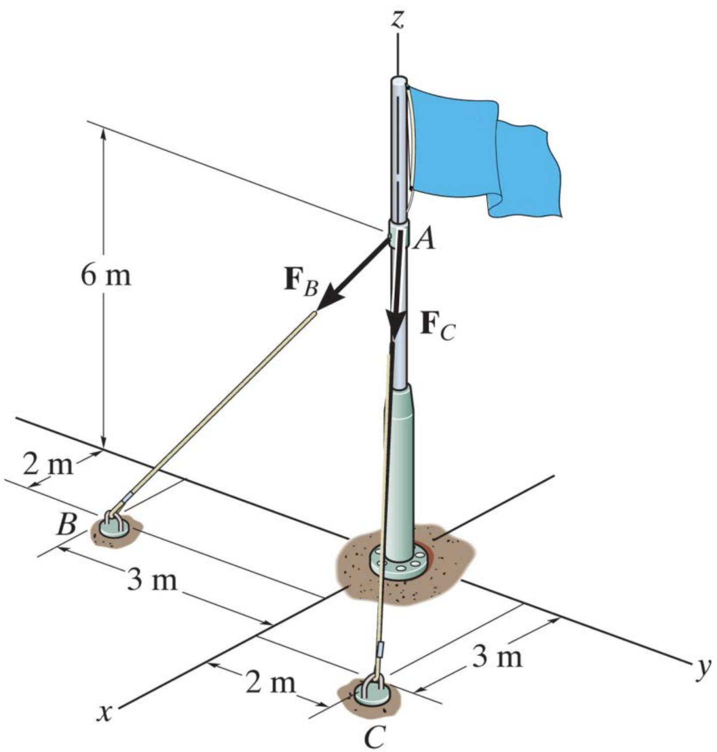
$= 1174.6$ N

$F_R = \underline{1175 \text{ N}}$

$\alpha = \cos^{-1}(460/1175) = \underline{66.9°}$

$\beta = \cos^{-1}(-40/1175) = \underline{92.0°}$

$\gamma = \cos^{-1}(-1080/1175) = \underline{157°}$

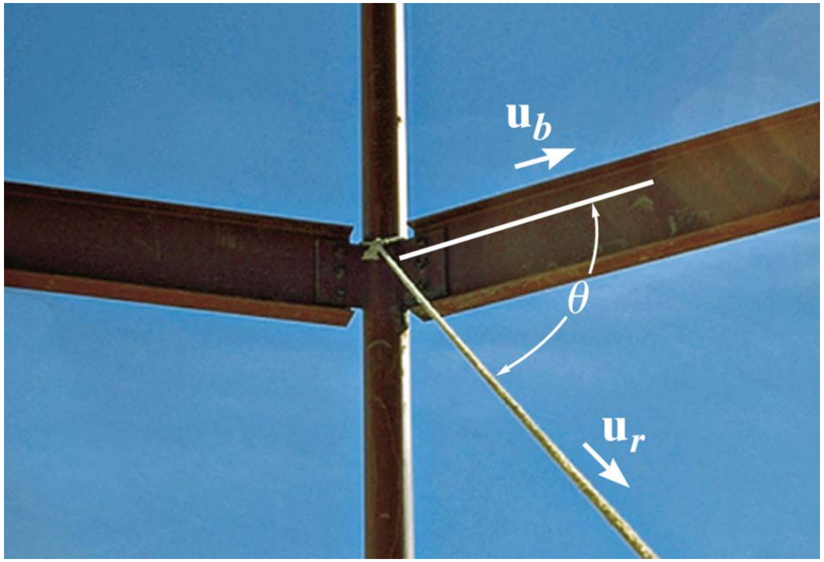


DOT PRODUCT

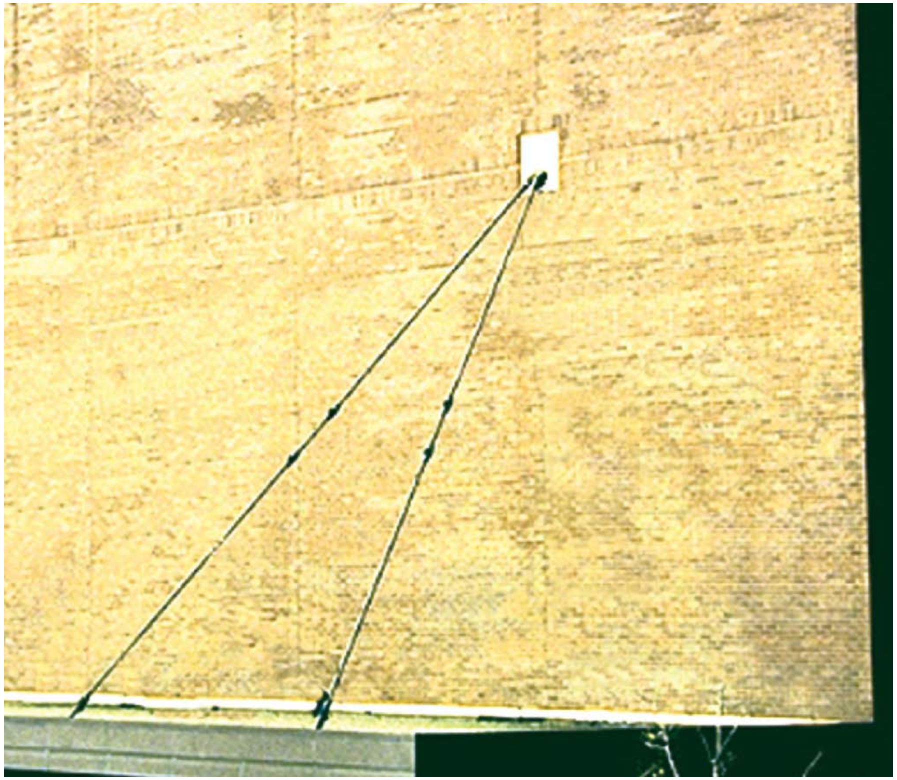
We use the vector dot product to:

a) determine an angle between two vectors and,

b) determine the projection of a vector along a specified line.

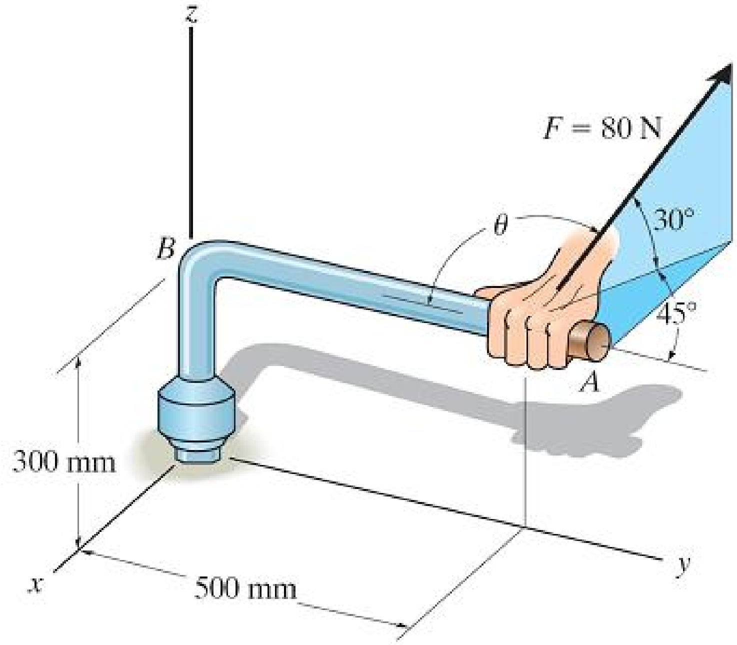


APPLICATIONS



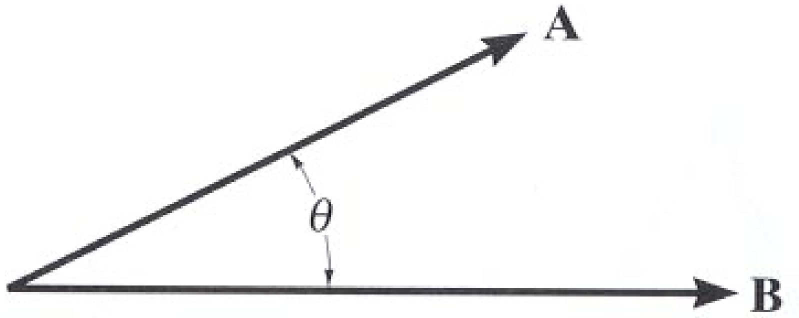
If you know the physical locations of the four cable ends, how could you calculate the angle between the cables at the common anchor?

APPLICATIONS (continued)



For the force ***F*** applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to arm AB of the pipe)?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A***•***B*** = A B cos θ.

The angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON (continued)

Examples: By definition, $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

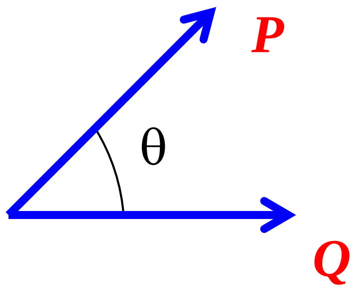
$\boldsymbol{i} \bullet \boldsymbol{i} = 1$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \bullet (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as



A) P Q sin θ   B) P Q cos θ

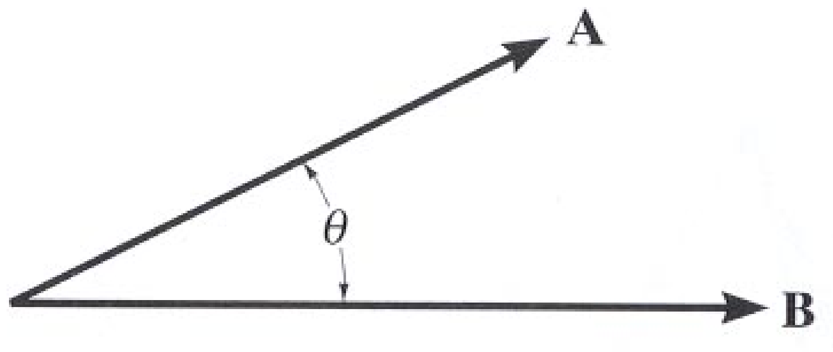
C) P Q tan θ   D) P Q sec θ

2. The dot product of two vectors results in a _________ quantity.

A) Scalar   B) Vector

C) Complex   D) Zero

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For these two vectors in Cartesian form, one can find the angle by

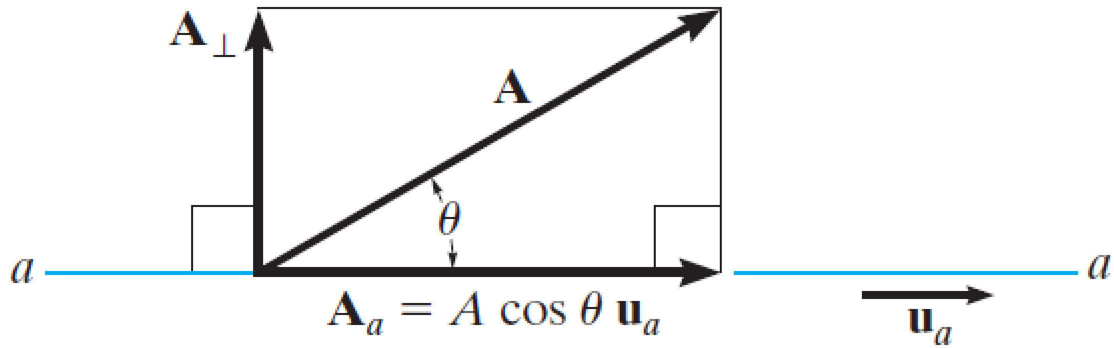
a) Find the dot product, $\mathbf{A} \bullet \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Find the magnitudes (A & B) of the vectors **A** & **B**, and

c) Use the definition of dot product and solve for θ, i.e.,

$\theta = \cos^{-1} [(\mathbf{A} \bullet \mathbf{B})/(A\ B)]$, where $0^\circ \leq \theta \leq 180^\circ$.

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{u_a}$ along line aa

2. Find the scalar projection of $\boldsymbol{A}$ along line aa by

$$A_{\|} = \boldsymbol{A} \bullet \boldsymbol{u_a} = A_x u_x + A_y u_y + A_z u_z$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{\parallel}$, by using the unit vector $\boldsymbol{u_a}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{\parallel} = A_{\parallel}\, \boldsymbol{u_a}$$

4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{\parallel}^2)^{1/2} \text{ and}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{\parallel}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{\parallel}$)

QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be _______________ to each other.
   A) Parallel (pointing in the same direction)
   B) Parallel (pointing in the opposite direction)
   C) Perpendicular
   D) Cannot be determined.

2. If a dot product of two unit vectors equals -1, then the vectors must be _________ to each other.
   A) Collinear but pointing in the opposite direction
   B) Parallel (pointing in the same direction)
   C) Perpendicular
   D) Cannot be determined.

QUIZ

3. The dot product can be used to find all of the following except _____ .

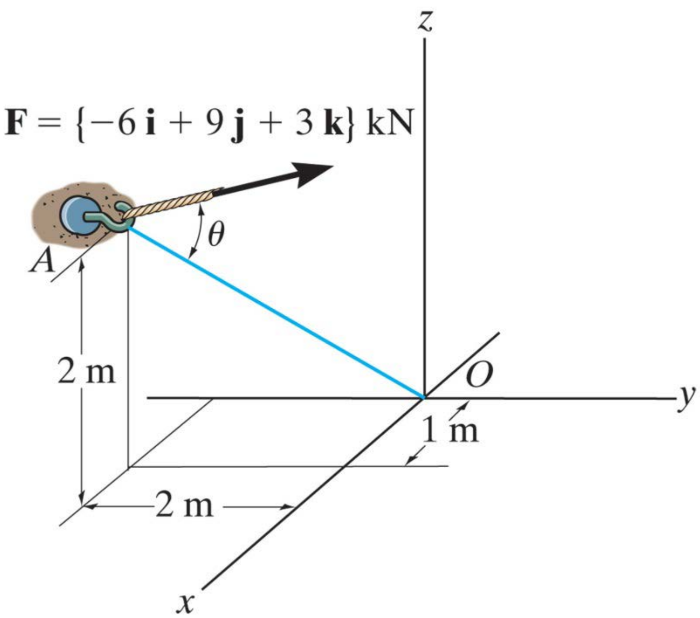
A) sum of two vectors

B) angle between two vectors

C) component of a vector parallel to another line

D) component of a vector perpendicular to another line

EXAMPLE VIII



Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

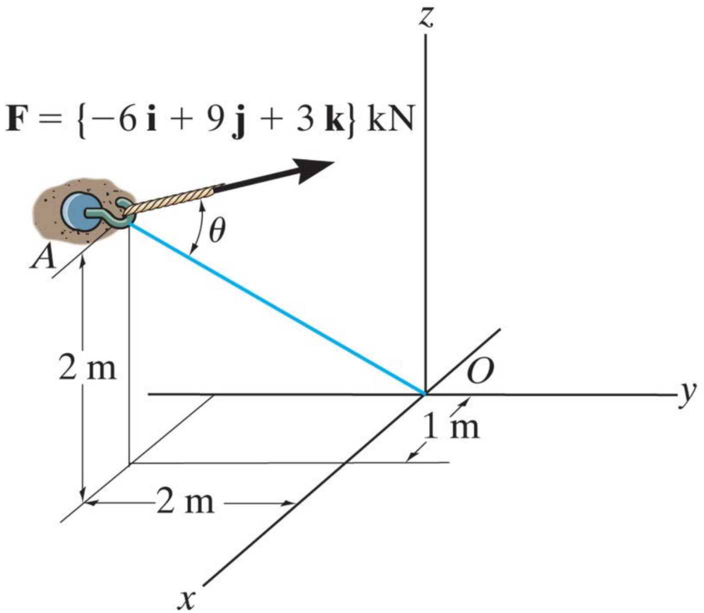
Plan:

1. Find $\boldsymbol{r}_{AO}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F\ r_{AO})\}$

3. Find the projection via $F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO}$ (or $F \cos \theta$)

EXAMPLE VIII (continued)



$\boldsymbol{r}_{AO} = \{-1\,\boldsymbol{i} + 2\,\boldsymbol{j} - 2\,\boldsymbol{k}\}$ m

$r_{AO} = \{(-1)^2 + 2^2 + (-2)^2\}^{1/2} = 3$ m

$\boldsymbol{F} = \{-6\,\boldsymbol{i} + 9\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ kN

$F = \{(-6)^2 + 9^2 + 3^2\}^{1/2} = 11.22$ kN

$\boldsymbol{F} \bullet \boldsymbol{r}_{AO} = (-6)(-1) + (9)(2) + (3)(-2) = 18$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F\, r_{AO})\}$

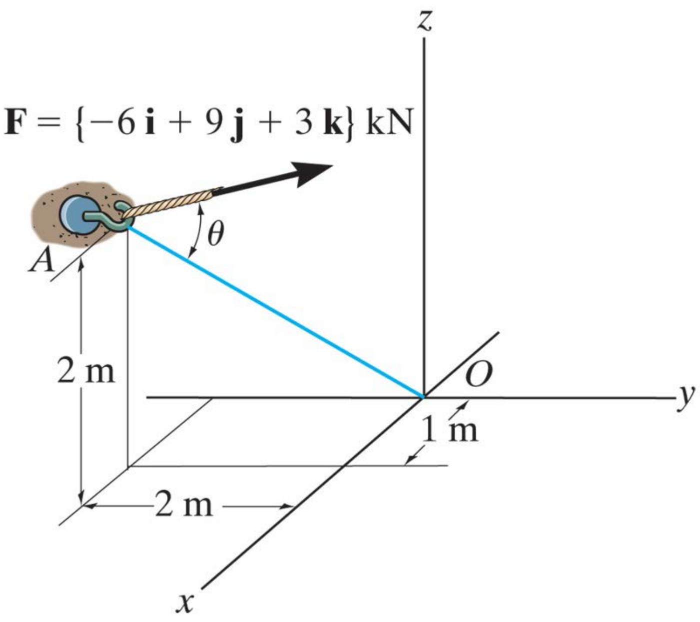
$\theta = \cos^{-1}\{18 / (11.22 \times 3)\} = \underline{57.67°}$

EXAMPLE VIII (continued)

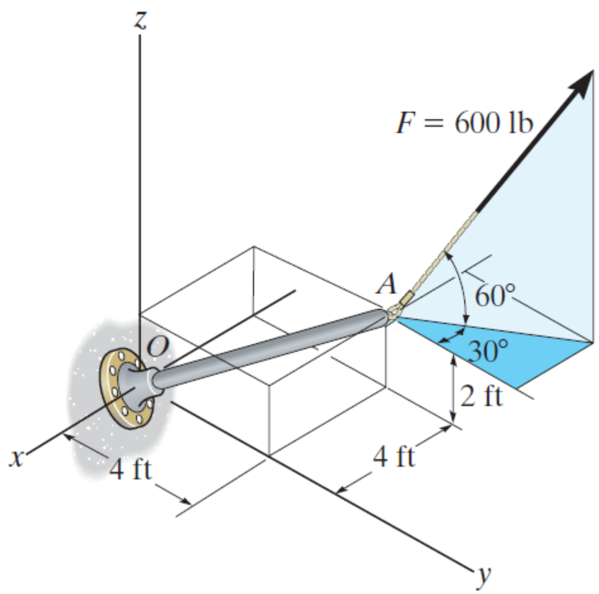
$\boldsymbol{u}_{AO} = \boldsymbol{r}_{AO} \,/\, \mathrm{r}_{\mathrm{AO}} = (-1/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} + (-2/3)\,\boldsymbol{k}$

$\mathrm{F}_{\mathrm{AO}} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO} = (-6)(-1/3) + (9)(2/3) + (3)(-2/3) = \underline{6.00 \text{ kN}}$

Or: $\mathrm{F}_{\mathrm{AO}} = \mathrm{F} \cos\theta = 11.22 \cos(57.67°) = \underline{6.00 \text{ kN}}$



EXAMPLE IX



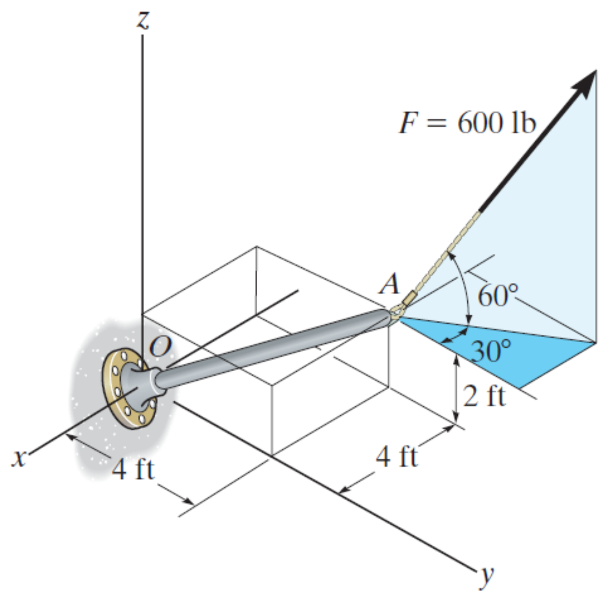
Given: The force acting on the pole at point A.

Find: Magnitude of the components of the force acting parallel and perpendicular to the axis of the pole.

Plan:

1. Find $\mathbf{F}$, $\mathbf{r}_{OA}$ and $\mathbf{u}_{OA}$
2. Determine the parallel component of $\mathbf{F}$ using $F_{\parallel} = \mathbf{F} \bullet \mathbf{u}_{OA}$
3. The perpendicular component of $\mathbf{F}$ is $F_{\perp} = (F^2 - F_{\parallel}^2)^{1/2}$

EXAMPLE IX (continued)



$\boldsymbol{r}_{OA} = \{-4\,\boldsymbol{i} + 4\,\boldsymbol{j} + 2\,\boldsymbol{k}\}$ ft

$r_{OA} = \{(-4)^2 + 4^2 + 2^2\}^{1/2} = 6$ ft

$\boldsymbol{u}_{OA} = -2/3\,\boldsymbol{i} + 2/3\,\boldsymbol{j} + 1/3\,\boldsymbol{k}$

$$\boldsymbol{F} = \{-(600 \cos 60°) \sin 30°\,\boldsymbol{i} + (600 \cos 60°) \cos 30°\,\boldsymbol{j} + (600 \sin 60°)\,\boldsymbol{k}\} \text{ lb}$$

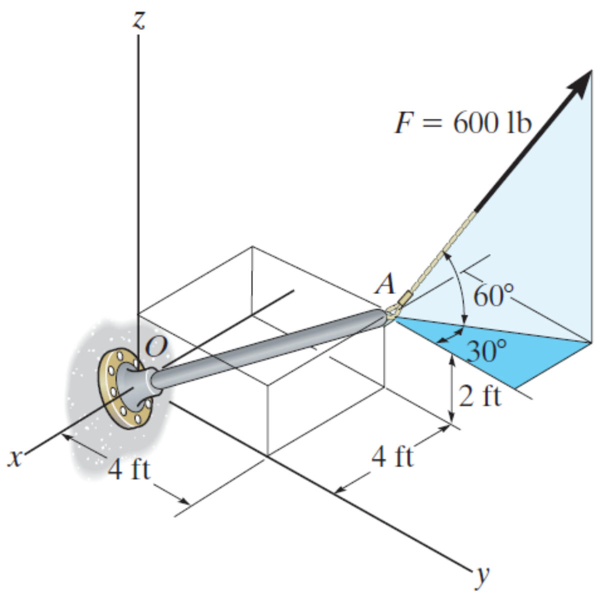
$\boldsymbol{F} = \{-150\,\boldsymbol{i} + 259.8\,\boldsymbol{j} + 519.6\,\boldsymbol{k}\}$ lb

The parallel component of ***F*** :

$F_{||} = \boldsymbol{F} \bullet \boldsymbol{u}_{OA} = \{-150\,\boldsymbol{i} + 259.8\,\boldsymbol{j} + 519.6\,\boldsymbol{k}\} \bullet \{-2/3\,\boldsymbol{i} + 2/3\,\boldsymbol{j} + 1/3\,\boldsymbol{k}\}$

$F_{||} = (-150) \times (-2/3) + 259.8 \times (2/3) + 519.6 \times (1/3) = \underline{446 \text{ lb}}$

EXAMPLE IX (continued)



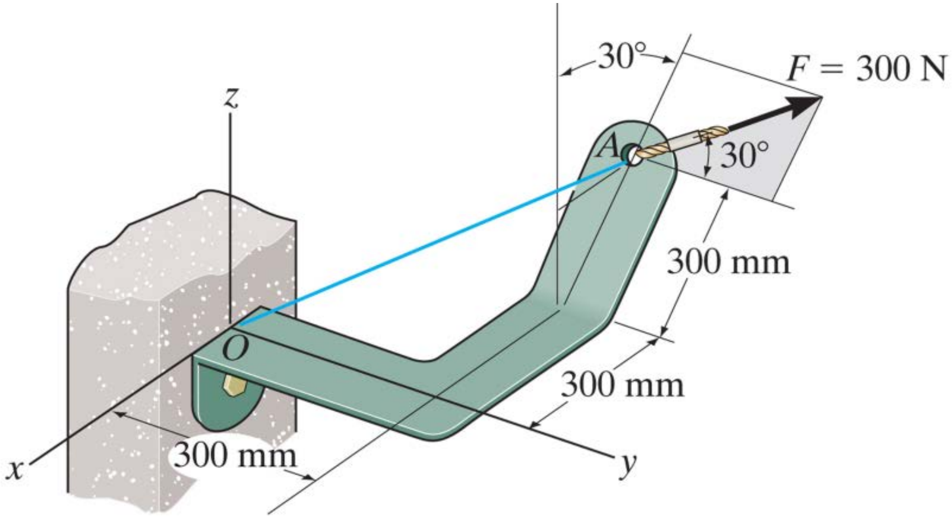
$F = 600$ lb

$F_{||} = 446$ lb

The perpendicular component of ***F*** :

$F_{\perp} = (F^2 - F_{||}^{\ 2})^{1/2} = (600^2 - 446^2)^{1/2} =$ 401 lb

EXAMPLE X



Given: The 300 N force acting on the bracket.

Find: The magnitude of the projected component of this force acting along line OA

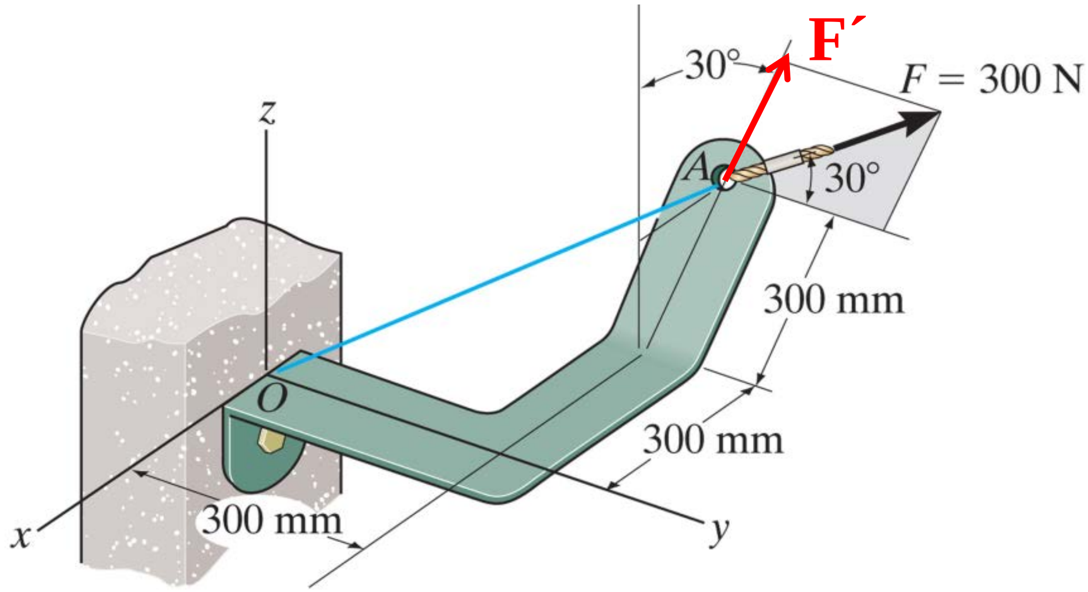
Plan:

1. Find $\boldsymbol{r}_{OA}$ and $\boldsymbol{u}_{OA}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{OA})/(F \times r_{OA})\}$

3. Then find the projection via $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u}_{OA}$ or $F\,(1)\cos\theta$

EXAMPLE X (continued)

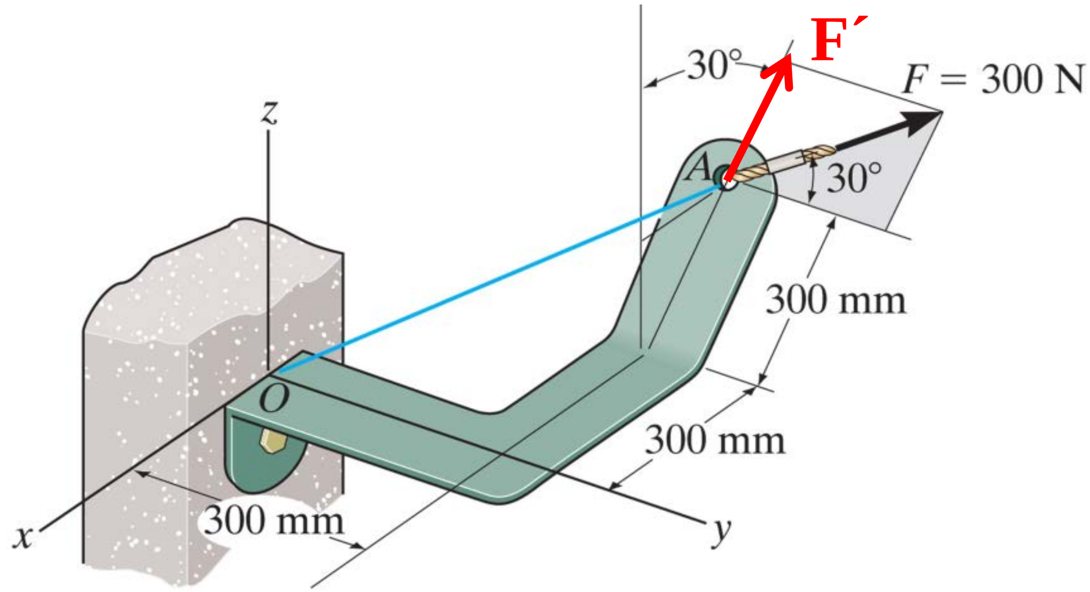


$$\boldsymbol{r}_{OA} = \{-0.450\ \boldsymbol{i} + 0.300\ \boldsymbol{j} + 0.260\ \boldsymbol{k}\}\ \text{m}$$

$$r_{OA} = \{(-0.450)^2 + 0.300^2 + 0.260^2\}^{1/2} = 0.60\ \text{m}$$

$$\boldsymbol{u}_{OA} = \boldsymbol{r}_{OA} / r_{OA} = \{-0.75\ \boldsymbol{i} + 0.50\ \boldsymbol{j} + 0.433\ \boldsymbol{k}\}$$

EXAMPLE X (continued)



$F' = 300 \sin 30° = 150$ N

$\boldsymbol{F} = \{-150 \sin 30°\boldsymbol{i} + 300 \cos 30°\boldsymbol{j} + 150 \cos 30°\boldsymbol{k}\}$ N

$\boldsymbol{F} = \{-75\ \boldsymbol{i} + 259.8\ \boldsymbol{j} + 129.9\ \boldsymbol{k}\}$ N

$F = \{(-75)^2 + 259.8^2 + 129.9^2\}^{1/2} = 300$ N

EXAMPLE X (continued)

The magnitude of the projected component of $\boldsymbol{F}$ along line OA will be

$$F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u}_{OA}$$
$$= (-75)(-0.75) + (259.8)\ (0.50) + (129.9)\ (0.433)$$
$$= \underline{242\ \ N}$$

OR

$$\boldsymbol{F} \bullet \boldsymbol{r}_{OA} = (-75)\ (-0.45) + (259.8)\ (0.30) + (129.9)\ (0.26)$$
$$= 145.5\ \ N{\cdot}m$$
$$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{OA})/(F \times r_{OA})\}$$
$$\theta = \cos^{-1}\{145.5\ /(300 \times 0.60)\} = \underline{36.1°}$$

$$F_{OA} = F \cos\theta = 300 \cos 36.1° = \underline{242\ N}$$